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Control of Marburg Virus Disease Spread in Humans under Hypersensitive Response through Fractal-Fractional

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Abstract:
The primary objective of this research is to examine the model of Marburg virus disease with therapy to prevent the infections from spreading throughout the community due to saliva, urine, and feces. A mathematical model is established using the created hypothesis for a healthy environment in order to examine the different rates of Marburg virus disease after taking control measures with treatment for different protection. The model is then converted into a fractional ordered model for continuous monitoring, including theoretical solutions, by using the Fractal Fractional operator. The model's global stability is studied through Lyapunov derivative by equilibrium and endemic point of model. Both qualitative and quantitative analysis is done on a proposed model with the power law kernel, with particular focus on unique solutions, positivity, and boundedness. By using of fixed point theory and the Lipschitz condition, there is only one exact solution. By confirming the Lyapunov function globally, equilibrium points that are both endemic and disease-free are checked for stability. Numerical simulations are utilized to investigate the effects of the fractal fractional operator on the generalized form of the power law kernel using a two-step Lagrange polynomial approach for ongoing monitoring of the Marburg virus disease and treatment strategies. The simulations show how different parameters affect the illness. In order to simulate the actual behavior and management of Marburg virus disease produced by saliva, urine, and feces and to implement control measures for a healthy environment with a hypersensitive reaction (HR), simulations have been constructed. This type of research will be helpful in figuring out how diseases spread and in developing management strategies based on our verified results for human in community.

Keywords: Mathematical Modeling; Boundedness and uniqueness; Sensitivity analysis; Marburg virus.

1. Introduction

Marburg virus disease (MVD) is a zoonotic illness with a case death rate of 88 percent. It is caused by the Marburg virus (MARV), An individual belonging to the Filoviridae family, which also harbors the Ebola virus [30]. Phylogenetic study of the genomic sequence data shows that there are five different lineages of MARV. These lineages are now considered to represent two distinct viruses, MARV and Ravn virus (RAVV), which are classified. A pleomorphic virus known as MARV has been found in six, round, U-shaped, rod-like, and most frequently filamentous in nature [35]. MARV virion typically have a diameter of 80 nm and an average length of 790 nm, though this can vary substantially [36]. Spikes that range in length from 5 to 10 nm and are positioned at intervals of roughly 10 nm protect the virion's surface [35, 37]. The hallmark of Marburg Virus Disease (MVD) is an acute hemorrhagic fever brought on by the Marburg [12]. Humans and various primates are occasionally exposed to the Marburg virus. It is found in Egyptian fruit bats (Rousettus aegyptiacus) naturally [12, 13, 14]. When laboratory employees in Yugoslavia's (now Serbia) Belgrade and Frankfurt, Germany contracted an infectious agent that was not previously identified, itwas August 1967 that the Marburg virus (MARV) first appeared. Thirty onepatients25 with initial infections and 6 with secondary infectionsdeveloped serious illness, which in seven of the cases led to death. A second case exhibiting illness symptoms was identified after the fact [1]. It was found that African green monkeys, or Chlorocebus aethiops, were the virus's primary carrier. They were shipped to each of the three locations after being imported from Uganda. Paradoxically, the monkey's kidney was the source of its initial infections. They had their cells extracted in order to create a vaccine against poliomyelitis. Scientists from Marburg and Hamburg worked together to isolate, characterize, and identify the causative agent in a remarkable period of fewer than three months [15, 16]. The Marburg virus (MARV) and the Ravn virus (RAVV), together known as the "marburg viruses" are naturally occurring reservoir hosts in Egyptian rousette bats (Rousettus aegyptiacus) instances of Marburg virus illness in humans (MVD) [2]. In environments such as homes or nosocomial hospitals, when infection control measures are not at their best, person-to-person transmission can happen. When individuals come into contact with infected bats shedding the virus during their invasion of caverns and other environments, the virus can spread straight from bats to people or from one person to another [3, 4]. Human-to-human transmission can occur through various means, particularly upon coming into contact with damaged skin or mucous membranes. The virus can be transmitted via blood or body fluids like feces, milk,sweat and others. Even in the absence of obvious symptoms, having sex with a guy who has just recovered from a Marburg virus attack whether orally, vaginally, or anallycarries a risk because possibly polluted seed.

Additionally, handling items such as medical equipment, needles, syringes, and personal belongings of infected individuals or corpses prior to burial, which may have been contaminated with bodily fluids, can lead to disease transmission [5, 6]. Even though Marburg was found more than fifty years ago.There are currently no approved preventive or therapeutic countermeasures. Since the Ebola outbreak in West Africa, more attention has been paid to Marburg in addition to Ebola [7]. The Marburg virus induces flu-like symptoms, including chills, a high fever (3940C), myalgia, extreme fatigue, and general malaise. Following these, patients may experience vomiting, severe nausea, abdominal pain, lack of appetite, and diarrhea that is watery. By days 4 to 5, additional symptoms may include pharyngitis, dysphasia, and an enanthem, along with a distinctive maculopapular rash. Other notable features of the infection are lymphadenopathy, thrombocytopenia, and leukopenia. Elevated fever neurological symptoms, including psychosis, agitation, aggressiveness, and encephalitis. abnormal vascular permeability, especially in the case of conjunctival injection, dyspnea, and edema. - Hemorrhaging symptoms include mucosal bleeding. Involvement of the liver, kidney, and pancreas. Reduced circulation from extreme dehydration, convulsions, restlessness, confusion, dementia, metabolic abnormalities, and multiple shock, organ failure, and coma. Prolonged convalescence; myalgia, fatigue, perspiration, skin peeling at rash areas, partial forgetfulness, and subsequent infections (non-fatal cases) [8].

Malaria, septicemia brought on by gram-negative bacteria, plague, leptospirosis, and typhoid should all be distinguished from MVD. Serum neutralization tests, electron microscopy, viral isolation, enzyme-linked immunogenetics assay (ELISA), and nucleic acid detection techniques like reverse transcriptase polymerase chain reactions (RT-PCR) are utilized for the MVD diagnosis. Unfortunately, these diagnostic methods are not widely accessible in many of the regions where MVD outbreaks are most prone to develop [9]. As of right now, there is only supportive care available for MVD. As the body increases its defenses against infection, these supporting actions prevent the disease from becoming more serious and leading to shock. Monoclonal antibodies for Ebola have been developed, and although there is currently no approved treatment for this virus, experts believe that in extreme cases, Marburg viral infections may be treated with them [10].

1.1. Preliminaries

Definition 1. ([23, 24]) Suppose U(t) be a non-necessarily differentiable function. Suppose $0 < \xi \leq 1$ and $0 < \psi \leq 1$, where ψ is fractal dimension and ξ is fractional order. A fractal fractional derivative having order $0 \leq \xi$, $\psi = 1$ is given for the power law kernel in the RiemannLiouville meaning..

$$ {}_{O}^{FFM}D_t^{\xi,\psi} f(t) = \frac{1}{\Gamma(1-\xi)} \frac{d}{dt^{\psi}} \int_0^t f(\Upsilon)(t-\Upsilon) d\Upsilon $$

Where

$$ \frac{df(t)}{dt^{\psi}} = \lim_{t \to t_1} \frac{f(t) - f(t_1)}{t^{2-\psi} - t_1^{2-\psi}} (2-\psi) $$

Definition 2: In the Riemann-Liouville concept, the fractal fractional derivative with the exponential decay kernel is given as [23, 24]

$$ {}_{O}^{FFM}D_t^{\xi,\psi} f(t) = \frac{M(\xi)}{1-\xi} \frac{d}{dt^{\psi}} \int_0^t f(\Upsilon) \exp[-\frac{\xi}{1-\xi}(t-\Upsilon)] d\Upsilon $$

in which $M(0) = 1 = M(1)$ and $M(\xi)$ denotes the normalization function.

Definition 3: In the Riemann-Liouville notion, the fractal-fractional derivative with Mittag-Leffler kernel is provided as [23, 24]

$$ {}_{O}^{FFM}D_t^{\xi,\psi} f(t) = \frac{AB(\xi)}{1-\xi} \frac{d}{dt^{\psi}} \int_0^t f(\Upsilon) E_{\xi}[-\frac{\xi}{1-\xi}(t-\Upsilon)^{\xi}] d\Upsilon $$

Where E_{ξ} is the Mittag-Leffler function and $AB(\xi) = 1 - \xi + \frac{\xi}{\Gamma(\xi)}$ is a normalization function.

Definition 4: [25] Think about $\Upsilon \in H^1(n_1, n_2), n_2 > n_1, n \in [0,1]$., subsequently the Caputo sense (ABC) Atangana-Baleanu derivative is defined as

$$ {}_{O}^{FFM}D_t^{\xi,\psi}[\Upsilon(t)] = \frac{M(\xi)}{1-\xi} \int_{n_1}^{\xi} \Upsilon(\acute{\psi}) E_{\xi}[\xi \frac{(t-\psi)^{\xi}}{\xi - 1}] d\psi $$

Where E_{ξ} is the one-perimeter Mittag-Leffler function and $M(\xi)$, $M(0) = M(1) = 1$, stands for the function of normalization.

Definition 5: [25] In relation to the normalizing function $M(\xi)$, the Attangana-Baleanu(ABC) The fractional integral can be found using

$$ {}_{O}^{FFM}I_t^{\xi,\psi} \Upsilon(t) = \frac{1-\xi}{M(\xi)} \Upsilon(t) + \frac{\xi}{\Gamma(\xi) M(\xi)} \int_{\xi_1}^{t} \Upsilon(\psi)(t-\psi)^{\xi-1} d\psi $$

2. Formulation of Marburg Virus

Our thesis presents an evolutionary compartmental model that explains the dynamics of Marburg transmission throughout two communities: African fruit bats, Rousettus aegyptiacus, and people. Furthermore, the human occupant is divided into various sections: the susceptible $S_p(t)$, exposed $E_p(t)$, infectious $I_p(t)$, and resurgent populations $R_p(t)$. Three categories, liable $S_a(t)$, contaminated $I_a(t)$, and restored $R_a(t)$, are employed to group the population of non-human species. The entry rate for Homosapien residents is $E_p.\lambda_p$ is the rate at which susceptible people become infected after coming into touch with an infected person or nonhuman object. This represents the human infection rate.

$$\lambda_p = \beta_{pa}\frac{I_a}{N_a} + \frac{\beta_{pp}(1+\theta\gamma(1-f))I_p}{N_p} \tag{1}$$

The effective contact rate is β_{pp}, Hence the likelihood that every interaction will result in a human being contaminated by an infectious non-human with an infected or recovered person multiplies the actual contact rate, which is β_{pa} will cause one human to become infected with the Marburg virus for every exchange. ω is the percentage of those exposed who develop extremely contaminated. In bat and human populations, the natural death rate is a and p, respectively.After coming into contact with a non-human that has previously been ill, the non-humans are added to the susceptible class at rate Λ_a, which is steady. They are subjected at a constant rate λ_a to the Marburg virus that is specified to

$$\lambda_a = \beta_{aa}\frac{I_a}{N_a}, \tag{2}$$

where β_{aa}, this denotes the real contact rate and shows the probability of spreading the infection to a bat with each touch. The animal's rate of recovery is ρ.

$$\begin{aligned}
S_p'(t) &= \Lambda_p - (\mu_p + \lambda_p)S_p(t),\\
E_p'(t) &= \lambda_p S_p(t) - (\mu_p + \omega)E_p(t),\\
I_p'(t) &= \omega E_p(t) - (\mu_p + \gamma)I_p(t),\\
R_p'(t) &= \gamma(1-f)I_p(t) - \mu_p R_p(t),\\
S_a'(t) &= \Lambda_a - (\mu_a + \lambda_a)S_a(t),\\
I_a'(t) &= \lambda_a S_a(t) - (\mu_a + \rho)I_a(t),\\
R_a'(t) &= \rho I_a(t) - \mu_a R_a(t),
\end{aligned} \tag{3}$$

by starting circumstance:

$S_p(0) \geq 0.$, $E_p(0) \geq 0.$,$I_p(0) \geq 0.$,$R_p(0) \geq 0.$,$S_a(0) \geq 0.$,$I_a(0) \geq 0.$,$R_a(0) \geq 0.$

$$\begin{aligned}
S_p(t) + E_p(t) + I_p(t) + R_p(t) &= N_p(t) and\\
S_a(t) + I_a(t) + R_a(t) &= N_a(t)
\end{aligned}$$

Utilizing the fractal fractional operator definition, we obtain

$$\begin{aligned}
{}^{FFM}_{O}D_t^{\xi,\psi} S_p(t) &= \Lambda_p - (\mu_p + \lambda_p)S_p(t),\\
{}^{FFM}_{O}D_t^{\xi,\psi} E_p(t) &= \lambda_p S_p(t) - (\mu_p + \omega)E_p(t),\\
{}^{FFM}_{O}D_t^{\xi,\psi} I_p(t) &= \omega E_p(t) - (\mu_p + \gamma)I_p(t),\\
{}^{FFM}_{O}D_t^{\xi,\psi} R_p(t) &= \gamma(1-f)I_p(t) - \mu_p R_p(t),\\
{}^{FFM}_{O}D_t^{\xi,\psi} S_a(t) &= \Lambda_a - (\mu_a + \lambda_a)S_a(t),\\
{}^{FFM}_{O}D_t^{\xi,\psi} I_a(t) &= \lambda_a S_a(t) - (\mu_a + \rho)I_a(t),\\
{}^{FFM}_{O}D_t^{\xi,\psi} R_a(t) &= \rho I_a(t) - \mu_a R_a(t),
\end{aligned} \tag{4}$$

Theorem.1 *Assume that the starting points are* $\{S_p(0), E_p(0), I_p(0), R_p(0), S_a(0), I_a(0), R_a(0)\} \subset Y$ Next, if the solutions $\{S_p, E_p, I_p, R_p, S_a, I_a, R_a\}$ exist, Each of these is +ve for all $t \geq 0.$.

Proof:

Let's start with a basic analysis that highlights real-world scenarios with positive numbers using the suggested method in order to show why the solutions are preferable [24, 29, 31]. We look at the requirements that guarantee positive results from the proposed paradigm. We are going to determine the accepted norm.

$$\| \alpha \|_\infty = \sup_{t \in D_\alpha} |\alpha(t)|, \tag{5}$$

where α's domain is denoted by D_α. First, let's look at the $S_p(t)$ class.

$$ {}_{O}^{FFM}D_t^{\xi,\psi} S_p(t) = \Lambda_p - (\mu_p + \lambda_p) S_p(t), \forall t \geq 0$$

$$ {}_{O}^{FFM}D_t^{\xi,\psi} S_p(t) \geq -(\mu_p + \lambda_p) S_p(t), \forall t \geq 0$$

this yield,

$$S_p(t) \geq S_p(0) E_\xi [-\frac{c^{1-\psi}\xi(\mu_p + \lambda_p)t^\xi}{AB(\xi) - (1-\xi)(\mu_p + \lambda_p)}], \forall t \geq 0$$

C represents the component of time. This means $t \geq 0.$, $S_p(t)$ is +ive.

Now we have the function $E_p(t)$.

$$ {}_{O}^{FFM}D_t^{\xi,\psi} E_p(t) = \lambda_p S_p(t) - (\mu_p + \omega) E_p(t),$$

$$ {}_{O}^{FFM}D_t^{\xi,\psi} E_p(t) \geq -(\mu_p + \omega) E_p(t),$$

this yields that

$$E_p(t) \geq S_p(0) E_\xi [-\frac{c^{1-\psi}\xi(\mu_p + \omega)t^\xi}{AB(\xi) - (1-\xi)(\mu_p + \omega)}], \forall t \geq 0$$

C represents the component of time. This means $t \geq 0$, $E_p(t)$ is +ive.

Let's go on to the function $I_p(t)$.

$$ {}_{O}^{FFM}D_t^{\xi,\psi} I_p(t) = \omega E_p(t) - (\mu_p + \gamma) I_p(t),$$

$$ {}_{O}^{FFM}D_t^{\xi,\psi} I_p(t) \geq -(\mu_p + \gamma) I_p(t)$$

This results as

$$I_p(t) \geq I_p(0) E_\xi [-\frac{c^{1-\psi}\xi(\mu_p + \gamma)t^\xi}{AB(\xi) - (1-\xi)(\mu_p + \gamma)}], \forall t \geq 0$$

The time component is denoted by c. For any $t \geq 0$, this demonstrates that $I_p(t)$ is positive.

For $R_p(t)$,

$$ {}_{O}^{FFM}D_t^{\xi,\psi} R_p(t) = \gamma(1-f) I_p(t) - \mu_p R_p(t),$$

$$ {}_{O}^{FFM}D_t^{\xi,\psi} R_p(t) \geq -\mu_p R_p(t),$$

This implies that

$$R_p(t) \geq R_p(0)E_\xi[-\frac{c^{1-\psi}\xi(\mu_p)t^\xi}{AB(\xi)-(1-\xi)(\mu_p)}], \forall t \geq 0$$

The time component is denoted by c. This gives that for any $t \geq 0$, $R_p(t)$ is positive.

For $S_a(t)$,

$$\begin{aligned} {}_{O}^{FFM}D_t^{\xi,\psi}S_a(t) &= \Lambda_a - (\mu_a + \lambda_a)S_a(t), \end{aligned}$$

$${}_{O}^{FFM}D_t^{\xi,\psi}S_a(t) \geq -(\mu_a + \lambda_a)S_a(t),$$

This yields,

$$S_a(t) \geq S_a(0)E_\xi[-\frac{c^{1-\psi}\xi(\mu_a+\lambda_a)t^\xi}{AB(\xi)-(1-\xi)(\mu_a+\lambda_a)}], \forall t \geq 0$$

The time component is denoted by c. This gives that for any $t \geq 0$, $S_a(t)$ is positive.

For the function $I_a(t)$,

$$\begin{aligned} {}_{O}^{FFM}D_t^{\xi,\psi}I_a(t) &= \lambda_a S_a(t) - (\mu_a + \rho)I_a(t), \end{aligned}$$

$${}_{O}^{FFM}D_t^{\xi,\psi}I_a(t) \geq -(\mu_a + \rho)I_a(t),$$

$$I_a(t) \geq I_a(0)E_\xi[-\frac{c^{1-\psi}\xi(\mu_a+\rho)t^\xi}{AB(\xi)-(1-\xi)(\mu_a+\rho)}], \forall t \geq 0$$

The time component is denoted by c. This demonstrates that for any $t \geq 0$, $I_a(t)$ is positive.

Now for the function $R_a(t)$,

$$\begin{aligned} {}_{O}^{FFM}D_t^{\xi,\psi}R_a(t) &= \rho I_a(t) - \mu_a R_a(t), \end{aligned}$$

$${}_{O}^{FFM}D_t^{\xi,\psi}R_a(t) \geq -\mu_a R_a(t),$$

This results

$$R_a(t) \geq R_a(0)E_\xi[-\frac{c^{1-\psi}\xi(\mu_a)t^\xi}{AB(\xi)-(1-\xi)(\mu_a)}], \forall t \geq 0$$

The component of time is denoted by c. This demonstrates that for any $t \geq 0$, $R_a(t)$ is positive.

Theorem.2: *T*he proposed Marburg virus model, in addition to the original circumstances

$$\begin{aligned}
{}_{O}^{FFM}D_t^{\xi,\psi}S_p(t) &= \Lambda_p - (\mu_p + \lambda_p)S_p(t), \\
{}_{O}^{FFM}D_t^{\xi,\psi}E_p(t) &= \lambda_p S_p(t) - (\mu_p + \omega)E_p(t), \\
{}_{O}^{FFM}D_t^{\xi,\psi}I_p(t) &= \omega E_p(t) - (\mu_p + \gamma)I_p(t), \\
{}_{O}^{FFM}D_t^{\xi,\psi}R_p(t) &= \gamma(1-f)I_p(t) - \mu_p R_p(t), \\
{}_{O}^{FFM}D_t^{\xi,\psi}S_a(t) &= \Lambda_a - (\mu_a + \lambda_a)S_a(t), \\
{}_{O}^{FFM}D_t^{\xi,\psi}I_a(t) &= \lambda_a S_a(t) - (\mu_a + \rho)I_a(t), \\
{}_{O}^{FFM}D_t^{\xi,\psi}R_a(t) &= \rho I_a(t) - \mu_a R_a(t),
\end{aligned}$$

is restricted to and unique in R^7_+.

Proof: We employ the technique described in[32] , we got

$$
\begin{aligned}
{}^{FFM}_{O}D^{\xi,\psi}_{t}S_p(t)_{S_p=0} &= \Lambda_p \geq 0.,\\
{}^{FFM}_{O}D^{\xi,\psi}_{t}E_p(t)_{E_p=0} &= \lambda_p S_p(t) \geq 0.,\\
{}^{FFM}_{O}D^{\xi,\psi}_{t}I_p(t)_{I_p=0} &= \omega E_p(t) \geq 0.\\
{}^{FFM}_{O}D^{\xi,\psi}_{t}R_p(t)_{R_p=0} &= \gamma I_p(t) \geq 0.\\
{}^{FFM}_{O}D^{\xi,\psi}_{t}S_a(t)_{S_a=0} &= \Lambda_a \geq 0.,\\
{}^{FFM}_{O}D^{\xi,\psi}_{t}I_a(t)_{I_a=0} &= \lambda_a S_a(t) \geq 0.,\\
{}^{FFM}_{O}D^{\xi,\psi}_{t}R_a(t)_{R_a=0} &= \rho I_a(t) \geq 0., \qquad (6)
\end{aligned}
$$

If $\{S_p(0), E_p(0), I_p(0), R_p(0), S_a(0), I_a(0), R_a(0)\} \in R^7_+$ The answer cannot be able to escape the hyperline, as stated in (3.6). It demonstrates that the domain R^7_+ is an invariant set that is positive.

2.1. *Effect of global derivative*

The integral that appears the most frequently is the Riemann-Stieltjes integral, and one specific example is the classical integral, as the literature has long since demonstrated. If

$$Z(\alpha) = \int z(\alpha)d(\alpha) \qquad (7)$$

The Riemann-Stieltjes integral is

$$Z_w(\alpha) = \int z(\alpha)dw(\alpha) \qquad (8)$$

$z(\alpha)$ global derivative with respect to $w(\alpha)$. is

$$D_w z(\alpha) = \lim_{p\to 0} \frac{z(\alpha+h)-z(\alpha)}{w(\alpha+h)-w(\alpha)} \qquad (9)$$

If distinguishing between the two roles may be done in a traditional manner, then,

$$D_w z(\alpha) = \frac{z^{'}(\alpha)}{w^{'}(\alpha)} \qquad (10)$$

Given that $\forall\alpha \in D_{w^{'}}$ ', $w^{'}(\alpha) \neq 0$. We will use this concept in this part to see if it affects the Marburg virus model in any way. We use the global derivative in place of the classical derivative to accomplish this.

$$
\begin{aligned}
D_w S_p &= \Lambda_p - (\mu_p + \lambda_p)S_p(t),\\
D_w E_p &= \lambda_p S_p(t) - (\mu_p + \omega)E_p(t),\\
D_w I_p &= \omega E_p(t) - (\mu_p + \gamma)I_p(t),\\
D_w R_p &= \gamma(1-f)I_p(t) - \mu_p R_p(t),\\
D_w S_a &= \Lambda_a - (\mu_a + \lambda_a)S_a(t),\\
D_w I_a &= \lambda_a S_a(t) - (\mu_a + \rho)I_a(t),\\
D_w R_a &= \rho I_a(t) - \mu_a R_a(t), \qquad (11)
\end{aligned}
$$

To keep things simple, w is differentiable, thus

$$
\begin{aligned}
S_p^{'} &= w^{'}[\Lambda_p-(\mu_p+\lambda_p)S_p(t)],\\
E_p^{'} &= w^{'}[\lambda_p S_p(t)-(\mu_p+\omega)E_p(t)],\\
I_p^{'} &= w^{'}[\omega E_p(t)-(\mu_p+\gamma)I_p(t)],\\
R_p^{'} &= w^{'}[\gamma(1-f)I_p(t)-\mu_p R_p(t)],\\
S_a^{'} &= w^{'}[\Lambda_a-(\mu_a+\lambda_a)S_a(t)],\\
I_a^{'} &= w^{'}[\lambda_a S_a(t)-(\mu_a+\rho)I_a(t)],\\
R_a^{'} &= w^{'}[\rho I_a(t)-\mu_a R_a(t)],
\end{aligned}
\tag{12}
$$

The result of appropriately selecting the function $w(t)$ will be a certain process. For example, fractal behavior will be seen if $w(t)=t^{\alpha}$, where $\alpha \in R$. In light of the situation, we have

$$
\|w^{'}\|_{\infty} = \sup_{t\in D_{w^{'}}} |w^{'}(t)| < N \tag{13}
$$

The next example will demonstrate that there can only be one variable solution for the system of equations.

$$
\begin{aligned}
S_p^{'} &= w^{'}[\Lambda_p-(\mu_p+\lambda_p)S_p(t)] = Z_1(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
E_p^{'} &= w^{'}[\lambda_p S_p(t)-(\mu_p+\omega)E_p(t)] = Z_2(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
I_p^{'} &= w^{'}[\omega E_p(t)-(\mu_p+\gamma)I_p(t)] = Z_3(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
R_p^{'} &= w^{'}[\gamma(1-f)I_p(t)-\mu_p R_p(t)] = Z_4(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
S_a^{'} &= w^{'}[\Lambda_a-(\mu_a+\lambda_a)S_a(t)] = Z_5(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
I_a^{'} &= w^{'}[\lambda_a S_a(t)-(\mu_a+\rho)I_a(t)] = Z_6(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
R_a^{'} &= w^{'}[\rho I_a(t)-\mu_a R_a(t)] = Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),
\end{aligned}
\tag{14}
$$

To accomplish , We must confirm the next two requirements.

$$
1. |Z(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 < K(1+|S|^2),
$$

$$
\begin{aligned}
2. \forall S_{p1},S_{p2}, we have,, \|Z(t,S_{p1},E_p,I_p,R_p,S_a,I_a,R_a)-Z(t,S_{p2},E_p,I_p,R_p,S_a,I_a,R_a)\|^2\\
< \overline{K}\|S_{p1}-S_{p2}\|_{\infty}^2
\end{aligned}
$$

Initially,

$$
\begin{aligned}
|Z_1(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 &= |w^{'}[\Lambda_p-(\mu_p+\lambda_p)S_p(t)]|^2\\
&= |w^{'}[\Lambda_p+(-\mu_p-\lambda_p)S_p]|^2\\
&\leq 2|w^{'}|^2[|\Lambda_p|^2+|(-\mu_p-\lambda_p)S_p|^2]\\
&\leq 2\sup_{t\in D_{w^{'}}}|w^{'}|^2\Lambda_p^2+4\sup_{t\in D_{w^{'}}}|w^{'}|^2(\mu_p^2+\lambda_p^2)|S_p|^2\\
&\leq 2\|w^{'}\|_{\infty}^2\Lambda_p^2+4\|w^{'}\|_{\infty}^2(\mu_p^2+\lambda_p^2)|S_p|^2\\
&\leq 2\|w^{'}\|_{\infty}^2\Lambda_p^2\left(1+\frac{2(\mu_p^2+\lambda_p^2)|S_p|^2}{\Lambda_p^2}\right)\\
&\leq K_1(1+|S_p|^2)
\end{aligned}
$$

Under the condition

$$\frac{2(\mu_p^2+\lambda_p^2)}{\Lambda_p^2} \quad < \quad 1$$

Where

$$K_1 \quad = \quad 2\|w^{'}\|_\infty^2\Lambda_p^2$$

$$\begin{aligned}
|Z_2(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 &= |w^{'}[\lambda_p S_p(t)-(\mu_p+\omega)E_p(t)]|^2\\
&= |w^{'}[\lambda_p S_p(t)+(-\mu_p-\omega)E_p(t)]|^2\\
&\leq 2|w^{'}(|\lambda_p S_p(t)|^2+|(-\mu_p-\omega)E_p(t)|^2)\\
&\leq 2\|w^{'}\|_\infty^2\lambda_p^2\|S_p\|_\infty^2+4\|w^{'}\|_\infty^2(\mu_p^2+\omega^2)|E_p|^2\\
&\leq 2\|w^{'}\|_\infty^2\lambda_p^2\|S_p\|_\infty^2(1+\frac{2(\mu_p^2+\omega^2)|E_p|^2}{\lambda_p^2\|S_p\|_\infty^2})\\
&\leq K_2(1+|E_p|^2)
\end{aligned}$$

Under the condition,

$$\frac{2(\mu_p^2+\omega^2)|E_p|^2}{\lambda_p^2\|S_p\|_\infty^2}<1$$

Where,

$$K_2 \quad = \quad 2\|w^{'}\|_\infty^2\lambda_p^2\|S_p\|_\infty^2$$

$$\begin{aligned}
|Z_3(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 &= |w^{'}[\omega E_p(t)-(\mu_p+\gamma)I_p(t)]|^2\\
&= |w^{'}[\omega E_p(t)+(-\mu_p-\gamma)I_p(t)]|^2\\
&\leq 2|w^{'}|^2(|\omega E_p|^2+|(-\mu_p-\gamma)I_p|^2)\\
&\leq 2|w^{'}|^2\omega^2|E_p|^2+4|w^{'}|^2(\mu_p^2+\gamma^2)|I_p|^2\\
&\leq 2\|w^{'}\|_\infty^2\omega^2\|E_p\|_\infty^2+4\|w^{'}\|_\infty^2(\mu_p^2+\gamma^2)|I_p|^2\\
&\leq 2\|w^{'}\|_\infty^2\omega^2\|E_p\|_\infty^2(1+\frac{2(\mu_p^2+\gamma^2)|I_p|^2}{\omega^2\|E_p\|_\infty^2})\\
&\leq K_3(1+|I_p|^2)
\end{aligned}$$

Under the condition,

$$\frac{2(\mu_p^2+\gamma^2)}{\omega^2\|E_p\|_\infty^2}<1$$

Where,

$$K_3 \quad = \quad 2\|w^{'}\|_\infty^2\omega^2\|E_p\|_\infty^2$$

$$
\begin{aligned}
|Z_4(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 &= |w^{'}[\gamma(1-f)I_p(t)-\mu_pR_p(t)]|^2 \\
&= |w^{'}[(\gamma-\gamma f)I_p(t)-\mu_pR_p(t)]|^2 \\
&\leq 2|w^{'}|^2[|(\gamma-\gamma f)I_p(t)|^2+|-\mu_pR_p(t)|^2] \\
&\leq 2|w^{'}|^2[2(\gamma^2-\gamma^2f^2)|I_p|^2+\mu_p^2|R_p|^2] \\
&\leq 4|w^{'}|^2(\gamma^2-\gamma^2f^2)|I_p|^2+2|w^{'}|^2\mu_p^2|R_p|^2 \\
&\leq 4\sup_{t\in D_{w^{'}}}|w^{'}|^2(\gamma^2-\gamma^2f^2)\sup_{t\in D_{I_p}}|I_p|^2+2\sup_{t\in D_{w^{'}}}|w^{'}|^2\mu_p^2|R_p|^2 \\
&\leq 4\|w^{'}\|_\infty^2(\gamma^2-\gamma^2f^2)\|I_p\|_\infty^2+2\|w^{'}\|_\infty^2\mu_p^2|R_p|^2 \\
&\leq 4\|w^{'}\|_\infty^2(\gamma^2-\gamma^2f^2)\|I_p\|_\infty^2(1+\frac{\mu_p^2|R_p|^2}{2(\gamma^2-\gamma^2f^2)\|I_p\|_\infty^2}) \\
&\leq K_4(1+|R_p|^2)
\end{aligned}
$$

Under the condition,

$$\frac{\mu_p^2}{2(\gamma^2-\gamma^2f^2)\|I_p\|_\infty^2}<1$$

where,

$$K_4 = 4\|w^{'}\|_\infty^2(\gamma^2-\gamma^2f^2)\|I_p\|_\infty^2$$

$$
\begin{aligned}
|Z_5(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 &= |w^{'}[\Lambda_a-(\mu_a+\lambda_a)S_a(t)]|^2 \\
&= |w^{'}[\Lambda_a+(-\mu_a-\lambda_a)S_a(t)]|^2 \\
&\leq 2|w^{'}|^2[|\Lambda_a|^2+|(-\mu_a-\lambda_a)S_a(t)|^2] \\
&\leq 2\|w^{'}\|_\infty^2(\Lambda_a^2)+4\|w^{'}\|_\infty^2(\mu_a^2+\lambda_a^2)|S_a|^2 \\
&\leq 2\|w^{'}\|_\infty^2(\Lambda_a^2)[1+\frac{2(\mu_a^2+\lambda_a^2)|S_a|^2}{\Lambda_a^2}] \\
&\leq K_5(1+|S_a|^2)
\end{aligned}
$$

Under the condition

$$\frac{2(\mu_a^2+\lambda_a^2)}{\Lambda_a^2}<1$$

Where

$$K_5 = 2\|w^{'}\|_\infty^2(\Lambda_a^2)$$

$$
\begin{aligned}
|Z_6(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 &= |w^{'}[\lambda_aS_a(t)-(\mu_a+\rho)I_a(t)]|^2 \\
&= |w^{'}[\lambda_aS_a(t)+(-\mu_a-\rho)I_a(t)]|^2 \\
&\leq 2|w^{'}|^2[|\lambda_aS_a|^2+|(-\mu_a-\rho)I_a|^2] \\
&\leq 2|w^{'}|^2\lambda_a^2|S_a|^2+4|w^{'}|^2(\mu_a^2+\rho^2)|I_a|^2 \\
&\leq 2\|w^{'}\|_\infty^2(\lambda_a^2)\|S_a\|_\infty^2+4\|w^{'}\|_\infty^2(\mu_a^2+\rho^2)|I_a|^2 \\
&\leq 2\|w^{'}\|_\infty^2(\lambda_a^2)\|S_a\|_\infty^2[1+\frac{2(\mu_a^2+\rho^2)|I_a|^2}{(\lambda_a^2)\|S_a\|_\infty^2}] \\
&\leq K_6[1+|I_a|^2]
\end{aligned}
$$

Under the condition

$$\frac{2(\mu_a^2+\rho^2)|I_a|^2}{(\lambda_a^2)\|S_a\|_\infty^2}<1$$

Where

$$K_6 \;=\; 2\|w^{'}\|_\infty^2(\lambda_a^2)\|S_a\|_\infty^2$$

$$\begin{aligned}
|Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)|^2 &= |w^{'}[\rho I_a(t)-\mu_a R_a(t)]|^2\\
&= |w^{'}[\rho I_a(t)+(-\mu_a)R_a(t)]|^2\\
&\leq 2|w^{'}|^2(|\rho I_a|^2+|(-\mu_a)R_a|^2)\\
&\leq 2|w^{'}|^2(\rho^2|I_a|^2+(\mu_a^2)|R_a|^2)\\
&\leq \sup_{t\in D_{w^{'}}}|w^{'}|^2(\rho^2\sup_{t\in D_{I_a}}+\mu_a^2|R_a|^2)\\
&\leq 2\|w^{'}\|_\infty^2(\rho^2\|I_a\|_\infty^2+\mu_a^2|R_a|^2)\\
&\leq 2\|w^{'}\|_\infty^2\rho^2\|I_a\|_\infty^2(1+\mu_a^2|R_a|^2\rho^2\|I_a\|_\infty^2)\\
&\leq K_7(1+|R_a|^2)
\end{aligned}$$

Under the condition

$$\frac{\mu_a^2}{\rho^2\|I_a\|_\infty^2}<1$$

Where

$$K_7 \;=\; 2\|w^{'}\|_\infty^2\rho^2\|I_a\|_\infty^2$$

Consequently, the need for linear growth is met. In addition, we verify the Lipschitz condition. If

$$\begin{aligned}
&|Z_1(t,S_{p_1},E_p,I_p,R_p,S_a,I_a,R_a)-Z_1(t,S_{p_2},E_p,I_p,R_p,S_a,I_a,R_a)|^2\\
&\qquad = |w^{'}[-(\mu_p+\lambda_p)(S_{p_1}-S_{p_2}|^2]
\end{aligned}$$

$$\begin{aligned}
&|Z_1(t,S_{p_1},E_p,I_p,R_p,S_a,I_a,R_a)-Z_1(t,S_{p_2},E_p,I_p,R_p,S_a,I_a,R_a)|^2\\
&\qquad \leq |w^{'}|^2(2\mu_p^2+2\lambda_p^2)|(S_{p_1}-S_{p_2})|^2
\end{aligned}$$

$$\begin{aligned}
&\sup_{t\in D_{S_p}}|Z_1(t,S_{p_1},E_p,I_p,R_p,S_a,I_a,R_a)-Z_1(t,S_{p_2},E_p,I_p,R_p,S_a,I_a,R_a)|^2\\
&\qquad \leq \sup_{t\in D_{w^{'}}}|w^{'}|^2(2\mu_p^2+2\lambda_p^2)\sup_{t\in D_{S_p}}|(S_{p_1}-S_{p_2})|^2
\end{aligned}$$

$$\begin{aligned}
&\|Z_1(t,S_{p_1},E_p,I_p,R_p,S_a,I_a,R_a)-Z_1(t,S_{p_2},E_p,I_p,R_p,S_a,I_a,R_a)\|_\infty^2\\
&\qquad \leq \|w^{'}\|_\infty^2(2\mu_p^2+2\lambda_p^2)\|(S_{p_1}-S_{p_2})\|_\infty^2
\end{aligned}$$

$$\begin{aligned}
&\|Z_1(t,S_{p_1},E_p,I_p,R_p,S_a,I_a,R_a)-Z_1(t,S_{p_2},E_p,I_p,R_p,S_a,I_a,R_a)\|_\infty^2\\
&\qquad \leq \overline{K_1}\|(S_{p_1}-S_{p_2})\|_\infty^2
\end{aligned}$$

Where

$$\overline{K_1} \quad = \quad \|w^{'}\|_\infty^2(2\mu_p^2+2\lambda_p^2)$$

$$|Z_2(t,S_p,E_{p_1},I_p,R_p,S_a,I_a,R_a)-Z_2(t,S_p,E_{p_2},I_p,R_p,S_a,I_a,R_a)|^2 \\ = |w^{'}[-(\mu_p-\omega)(E_{p_1}-E_{p_2})]|^2$$

$$|Z_2(t,S_p,E_{p_1},I_p,R_p,S_a,I_a,R_a)-Z_2(t,S_p,E_{p_2},I_p,R_p,S_a,I_a,R_a)|^2 \\ \leq |w^{'}|^2(2\mu_p^2+2\omega^2)|(E_{p_1}-E_{p_2})|^2$$

$$\sup_{t\in D_{E_p}} |Z_2(t,S_p,E_{p_1},I_p,R_p,S_a,I_a,R_a)-Z_2(t,S_p,E_{p_2},I_p,R_p,S_a,I_a,R_a)|^2 \\ \leq \sup_{t\in D_{w^{'}}} |w^{'}|^2(2\mu_p^2+2\omega^2) \sup_{t\in D_{E_p}} |(E_{p_1}-E_{p_2})|^2$$

$$\|Z_2(t,S_p,E_{p_1},I_p,R_p,S_a,I_a,R_a)-Z_2(t,S_p,E_{p_2},I_p,R_p,S_a,I_a,R_a)\|_\infty^2 \\ \leq \|w^{'}\|_\infty^2(2\mu_p^2+2\omega^2)\|(E_{p_1}-E_{p_2})\|_\infty^2$$

$$\|Z_2(t,S_p,E_{p_1},I_p,R_p,S_a,I_a,R_a)-Z_2(t,S_p,E_{p_2},I_p,R_p,S_a,I_a,R_a)\|_\infty^2 \\ \leq \overline{K_2}\|(E_{p_1}-E_{p_2})\|_\infty^2$$

Where

$$\overline{K_2} \quad = \quad \|w^{'}\|_\infty^2(2\mu_p^2+2\omega^2)$$

$$|Z_3(t,S_p,E_p,I_{p_1},R_p,S_a,I_a,R_a)-Z_3(t,S_p,E_p,I_{p_2},R_p,S_a,I_a,R_a)|^2 \\ = |w^{'}[-(\mu_p+\gamma)(I_{p_1}-I_{p_2})]|^2$$

$$|Z_3(t,S_p,E_p,I_{p_1},R_p,S_a,I_a,R_a)-Z_3(t,S_p,E_p,I_{p_2},R_p,S_a,I_a,R_a)|^2 \\ \leq |w^{'}|^2(2\mu_p^2+2\gamma^2)|(I_{p_1}-I_{p_2})|^2$$

$$\sup_{t\in D_{I_p}} |Z_3(t,S_p,E_p,I_{p_1},R_p,S_a,I_a,R_a)-Z_3(t,S_p,E_p,I_{p_2},R_p,S_a,I_a,R_a)|^2 \\ \leq \sup_{t\in D_{w^{'}}} |w^{'}|^2(2\mu_p^2+2\gamma^2) \sup_{t\in D_{I_p}} |(I_{p_1}-I_{p_2})|^2$$

$$\|Z_3(t,S_p,E_p,I_{p_1},R_p,S_a,I_a,R_a)-Z_3(t,S_p,E_p,I_{p_2},R_p,S_a,I_a,R_a)\|_\infty^2 \\ \leq \|w^{'}\|_\infty^2(2\mu_p^2+2\gamma^2)\|(I_{p_1}-I_{p_2})\|_\infty^2$$

$$\|Z_3(t,S_p,E_p,I_{p_1},R_p,S_a,I_a,R_a)-Z_3(t,S_p,E_p,I_{p_2},R_p,S_a,I_a,R_a)\|_\infty^2 \\ \leq \overline{K_3}\|(I_{p_1}-I_{p_2})\|_\infty^2$$

Where

$$\overline{K_3} = \|w^{'}\|_\infty^2(2\mu_p^2+2\gamma^2)$$

$$|Z_4(t,S_p,E_p,I_p,R_{p_1},S_a,I_a,R_a)-Z_4(t,S_p,E_p,I_p,R_{p_2},S_a,I_a,R_a)|^2$$
$$=|w^{'}[-\mu_p(R_{p_1}-R_{p_2})]|^2$$

$$|Z_4(t,S_p,E_p,I_p,R_{p_1},S_a,I_a,R_a)-Z_4(t,S_p,E_p,I_p,R_{p_2},S_a,I_a,R_a)|^2$$
$$\leq |w^{'}|^2\mu_p^2|(R_{p_1}-R_{p_2})|^2$$

$$\sup_{t\in D_{R_p}}|Z_4(t,S_p,E_p,I_p,R_{p_1},S_a,I_a,R_a)-Z_4(t,S_p,E_p,I_p,R_{p_2},S_a,I_a,R_a)|^2$$
$$\leq \sup_{t\in D_{w^{'}}}|w^{'}|^2\mu_p^2\sup_{t\in D_{R_p}}|(R_{p_1}-R_{p_2})|^2$$

$$\|Z_4(t,S_p,E_p,I_p,R_{p_1},S_a,I_a,R_a)-Z_4(t,S_p,E_p,I_p,R_{p_2},S_a,I_a,R_a)\|_\infty^2$$
$$\leq \|w^{'}\|_\infty^2\mu_p^2\|(I_{p_1}-I_{p_2})\|_\infty^2$$

$$\|Z_4(t,S_p,E_p,I_p,R_{p_1},S_a,I_a,R_a)-Z_4(t,S_p,E_p,I_p,R_{p_2},S_a,I_a,R_a)\|_\infty^2$$
$$\leq \overline{K_4}\|(R_{p_1}-R_{p_2})\|_\infty^2$$

Where

$$\overline{K_4} = \|w^{'}\|_\infty^2\mu_p^2$$

$$|Z_5(t,S_p,E_p,I_p,R_p,S_{b_1},I_a,R_a)-Z_5(t,S_p,E_p,I_p,R_p,S_{b_2},I_a,R_a)|^2$$
$$=|w^{'}[-(\mu_a+\lambda a)(S_{b_1}-S_{b_2})]|^2$$

$$|Z_5(t,S_p,E_p,I_p,R_p,S_{b_1},I_a,R_a)-Z_5(t,S_p,E_p,I_p,R_p,S_{b_2},I_a,R_a)|^2$$
$$\leq |w^{'}|^2(2\mu_a^2+2\lambda_a^2)|(S_{b_1}-S_{b_2})|^2$$

$$\sup_{t\in D_{S_a}}|Z_5(t,S_p,E_p,I_p,R_p,S_{b_1},I_a,R_a)-Z_5(t,S_p,E_p,I_p,R_p,S_{b_2},I_a,R_a)|^2$$
$$\leq \sup_{t\in D_{w^{'}}}|w^{'}|^2(2\mu_a^2+2\lambda_a^2)\sup_{t\in D_{S_a}}|(S_{b_1}-S_{b_2})|^2$$

$$\|Z_5(t,S_p,E_p,I_p,R_p,S_{b_1},I_a,R_a)-Z_5(t,S_p,E_p,I_p,R_p,S_{b_2},I_a,R_a)\|_\infty^2$$
$$\leq \|w^{'}\|_\infty^2(2\mu_a^2+2\lambda_a^2)\|(S_{b_1}-S_{b_2})\|_\infty^2$$

$$\|Z_5(t,S_p,E_p,I_p,R_p,S_{b_1},I_a,R_a)-Z_5(t,S_p,E_p,I_p,R_p,S_{b_2},I_a,R_a)\|_\infty^2$$
$$\leq \overline{K_5}\|(S_{b_1}-S_{b_2})\|_\infty^2$$

Where

$$\overline{K_5} \quad = \quad \|w^{'}\|_{\infty}^{2}(2\mu_a^2 + 2\lambda_a^2)$$

$$|Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_1},R_a) - Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_2},R_a)|^2 = |w^{'}[-(\mu_a + \rho)(I_{b_1} - I_{b_2})]|^2$$

$$|Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_1},R_a) - Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_2},R_a)|^2 \leq |w^{'}|^2(2\mu_a^2 + 2\rho^2)|(I_{b_1} - I_{b_2})|^2$$

$$\sup_{t\in D_{I_a}} |Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_1},R_a) - Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_2},R_a)|^2 \leq \sup_{t\in D_{w^{'}}} |w^{'}|^2(2\mu_a^2 + 2\rho^2) \sup_{t\in D_{I_a}} |(I_{b_1} - I_{b_2})|^2$$

$$\|Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_1},R_a) - Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_2},R_a)\|_{\infty}^2 \leq \|w^{'}\|_{\infty}^2(2\mu_a^2 + 2\rho^2)\|(I_{b_1} - I_{b_2})\|_{\infty}^2$$

$$\|Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_1},R_a) - Z_6(t,S_p,E_p,I_p,R_p,S_a,I_{b_2},R_a)\|_{\infty}^2 \leq \overline{K_6}\|(I_{b_1} - I_{b_2})\|_{\infty}^2$$

Where

$$\overline{K_6} \quad = \quad \|w^{'}\|_{\infty}^{2}(2\mu_a^2 + 2\rho^2)$$

$$|Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_1}) - Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_2})|^2 = |w^{'}[-(\mu_a)(R_{b_1} - R_{b_2})]|^2$$

$$|Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_1}) - Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_2})|^2 \leq |w^{'}|^2(2\mu_a^2)|(R_{b_1} - R_{b_2})|^2$$

$$\sup_{t\in D_{R_a}} |Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_1}) - Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_2})|^2 \leq \sup_{t\in D_{w^{'}}} |w^{'}|^2(2\mu_a^2) \sup_{t\in D_{R_a}} |(R_{b_1} - R_{b_2})|^2$$

$$\|Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_1}) - Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_2})\|_{\infty}^2 \leq \|w^{'}\|_{\infty}^2(2\mu_a^2)\|(R_{b_1} - R_{b_2})\|_{\infty}^2$$

$$\|Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_1}) - Z_7(t,S_p,E_p,I_p,R_p,S_a,I_a,R_{b_2})\|_{\infty}^2 \leq \overline{K_7}\|(R_{b_1} - R_{b_2})\|_{\infty}^2$$

Where

$$\overline{K_7} \quad = \quad \|w^{'}\|_{\infty}^{2}(2\mu_a^2)$$

Then, system 3.4 has a unique solution under the given conditions.

$$\max[\frac{2(\mu_p^2 + \lambda_p^2)}{\Lambda_p^2}, \frac{2(\mu_p^2 + \omega^2)|E_p|^2}{\lambda_p^2\|S_p\|_{\infty}^2}, \frac{2(\mu_p^2 + \gamma^2)}{\omega^2\|E_p\|_{\infty}^2}, \frac{\mu_p^2}{2(\gamma^2 - \gamma^2 f^2)\|I_p\|_{\infty}^2}, \frac{2(\mu_a^2 + \lambda_a^2)}{\Lambda_a^2}, \frac{2(\mu_a^2 + \rho^2)|I_a|^2}{(\lambda_a^2)\|S_a\|_{\infty}^2}, \frac{\mu_a^2}{\rho^2\|I_a\|_{\infty}^2}] < 1$$

2.2. Global stability analysis

It is demonstrated that the global stability analysis provides the prerequisite for the elimination of sickness using Lyapunov's method and Lasalle's invariance principle.

2.2.1. First derivative of Lyapunov

Theorem 3:([33]) The equilibrium points that are endemic to the $S_p, E_p, I_p, R_p, S_a, I_a, R_a$ Global asymptotic stability of models is achieved when $R_0 > 1$ for the number of reproductions.

Proof. The function of Lyapunov can be expressed as follows for proof:

$$\begin{aligned} L(S_p^*, E_p^*, I_p^*, R_p^*, S_a^*, I_a^*, R_a^*) &= (S_p - S_p^* - S_p^* \log \frac{S_p^*}{S_p}) + (E_p - E_p^* - E_p^* \log \frac{E_p^*}{E_p}) \\ &\quad + (I_p - I_p^* - I_p^* \log \frac{I_p^*}{I_p}) + (R_p - R_p^* - R_p^* \log \frac{R_p^*}{R_p}) \\ &\quad + (S_a - S_a^* - S_a^* \log \frac{S_a^*}{S_a}) + (I_a - I_a^* - I_a^* \log \frac{I_a^*}{I_a}) \\ &\quad + (R_a - R_a^* - R_a^* \log \frac{R_a^*}{R_a}) \end{aligned} \tag{15}$$

Consequently, if the derivative is applied with respectto "t" on both sides, the result is

$$\begin{aligned} \frac{dL}{dt} = \dot{L} &= (\frac{S_p - S_p^*}{S_p})\dot{S_p} + (\frac{E_p - E_p^*}{E_p})\dot{E_p} + (\frac{I_p - I_p^*}{I_p})\dot{I_p} + (\frac{R_p - R_p^*}{R_p})\dot{R_p} \\ &\quad + (\frac{S_a - S_a^*}{S_a})\dot{S_a} + (\frac{I_a - I_a^*}{I_a})\dot{I_a} + (\frac{R_a - R_a^*}{R_a})\dot{R_a} \end{aligned} \tag{16}$$

$$\begin{aligned} \frac{dL}{dt} &= (\frac{S_p - S_p^*}{S_p})(\Lambda_p - (\mu_p + \lambda_p)S_p) + (\frac{E_p - E_p^*}{E_p})(\lambda_p S_p - (\mu_p + \omega)E_p) \\ &\quad + (\frac{I_p - I_p^*}{I_p})(\omega E_p - (\mu_p + \gamma)I_p) + (\frac{R_p - R_p^*}{R_p})(\gamma(1-f)I_p - \mu_p R_p) \\ &\quad + (\frac{S_a - S_a^*}{S_a})(\Lambda_a - (\mu_a + \lambda_a)S_a) + (\frac{I_a - I_a^*}{I_a})(\lambda_a S_a - (\mu_a + \rho)I_a) \\ &\quad + (\frac{R_a - R_a^*}{R_a})(\rho I_a - \mu_a R_a) \end{aligned} \tag{17}$$

Putting $S_p = S_p - S_p^*, E_p = E_p - E_p^*, I_p = I_p - I_p^*, R_p = R_p - R_p^*, S_a = S_a - S_a^*,$
$I_a = I_a - I_a^*, R_a = R_a - R_a^*$ leads to

$$\begin{aligned} \frac{dL}{dt} &= (\frac{S_p - S_p^*}{S_p})(\Lambda_p - (\mu_p + \lambda_p)(S_p - S_p^*)) \\ &\quad + (\frac{E_p - E_p^*}{E_p})(\lambda_p (S_p - S_p^*) - (\mu_p + \omega)(E_p - E_p^*)) \\ &\quad + (\frac{I_p - I_p^*}{I_p})(\omega(E_p - E_p^*) - (\mu_p + \gamma)(I_p - I_p^*)) \\ &\quad + (\frac{R_p - R_p^*}{R_p})(\gamma(1-f)(I_p - I_p^*) - \mu_p(R_p - R_p^*)) \\ &\quad + (\frac{S_a - S_a^*}{S_a})(\Lambda_a - (\mu_a + \lambda_a)(S_a - S_a^*)) \\ &\quad + (\frac{I_a - I_a^*}{I_a})(\lambda_a(S_a - S_a^*) - (\mu_a + \rho)(I_a - I_a^*)) \\ &\quad + (\frac{R_a - R_a^*}{R_a})(\rho(I_a - I_a^*) - \mu_a(R_a - R_a^*)) \end{aligned} \tag{18}$$

The above can be arranged as follows:

$$\begin{aligned}
\frac{dL}{dt} &= \Lambda_p - \Lambda_p \frac{S_p^*}{S_p} - (\mu_p + \lambda_p)\frac{(S_p - S_p^*)^2}{S_p} + \lambda_p S_p - \lambda_p S_p^* - \lambda_p \frac{S_p E_p^*}{E_p} + \lambda_p \frac{S_p^* E_p^*}{E_p} \\
&\quad -(\mu_p + \omega)\frac{(E_p - E_p^*)^2}{E_p} + \omega E_p - \omega \frac{E_p I_p^*}{I_p} - \omega E_p^* + \omega \frac{E_p^* I_p^*}{I_p} - (\mu_p + \gamma)\frac{(I_p - I_p^*)^2}{I_p} \\
&\quad +\gamma I_p - \gamma \frac{I_p R_p^*}{R_p} - \gamma I_p^* + \gamma \frac{I_p^* R p^*}{R_p} - \gamma f I_p + \gamma f \frac{I_p R_p^*}{R_p} + \gamma f I_p^* - \gamma f \frac{I_p^* R_p^*}{R_p} \\
&\quad -\mu_p \frac{(R_p - R_p^*)^2}{R_p} + \Lambda_a - \Lambda_a \frac{S_a^*}{S_a} - (\mu_a + \lambda_a)\frac{(S_a - S_a^*)^2}{S_a} + \lambda_a S_a - \lambda_a S_a^* - \lambda_a \frac{S_a I_a^*}{I_a} \\
&\quad +\lambda_a \frac{S_a^* I_a^*}{I_a} - (\mu_a + \rho)\frac{(I_a - I_a^*)^2}{I_a} + \rho I_a - \rho I_a^* - \rho \frac{I_a R_a^*}{R_a} + \rho \frac{I_a^* R_a^*}{R_a} \\
&\quad -\mu_a \frac{(R_a - R_a^*)^2}{R_a} \qquad (19)
\end{aligned}$$

It is possible to write (3.19) as to eliminate the complexity.

$$\frac{dL}{dt} = \Sigma - \Omega \qquad (20)$$

where

$$\begin{aligned}
\Sigma &= \Lambda_p + \lambda_p S_p + \lambda_p \frac{S_p^* E_p^*}{E_p} + \omega E_p + \omega \frac{E_p^* I_p^*}{I_p} + \gamma I_p + \gamma \frac{I_p^* R p^*}{R_p} + \gamma f \frac{I_p R_p^*}{R_p} + \gamma f I_p^* \\
&\quad +\Lambda_a + \lambda_a S_a + \lambda_a \frac{S_a^* I_a^*}{I_a} + \rho I_a + \rho \frac{I_a^* R_a^*}{R_a}
\end{aligned}$$

and

$$\begin{aligned}
\Omega &= +\Lambda_p \frac{S_p^*}{S_p} + (\mu_p + \lambda_p)\frac{(S_p - S_p^*)^2}{S_p} + \lambda_p S_p^* + \lambda_p \frac{S_p E_p^*}{E_p} \\
&\quad +(\mu_p + \omega)\frac{(E_p - E_p^*)^2}{E_p} + \omega \frac{E_p I_p^*}{I_p} + \omega E_p^* + (\mu_p + \gamma)\frac{(I_p - I_p^*)^2}{I_p} \\
&\quad +\gamma \frac{I_p R_p^*}{R_p} + \gamma I_p^* + \gamma f I_p + \gamma f \frac{I_p^* R_p^*}{R_p} + \mu_p \frac{(R_p - R_p^*)^2}{R_p} \\
&\quad +\Lambda_a \frac{S_a^*}{S_a} + (\mu_a + \lambda_a)\frac{(S_a - S_a^*)^2}{S_a} + \lambda_a S_a^* + \lambda_a \frac{S_a I_a^*}{I_a} \\
&\quad +(\mu_a + \rho)\frac{(I_a - I_a^*)^2}{I_a} + \rho I_a^* + \rho \frac{I_a R_a^*}{R_a} + \mu_a \frac{(R_a - R_a^*)^2}{R_a}
\end{aligned}$$

It is concluded that if $\Sigma < \Omega$, This yields $\frac{dL}{dt} < 0$, However when $S_p = S_p^*$, $E_p = E_p^*, I_p = I_p^*, R_p = R_p^*, S_a = S_a^*, I_a = I_a^*, R_a = R_a^*$

$$0 = \Sigma - \Omega \Rightarrow \frac{dL}{dt} = 0 \qquad (21)$$

It is evident that the proposed model's biggest compact invariant set in

$$\{(S_p^*, E_p^*, I_p^*, R_p^*, S_a^*, I_a^*, R_a^*) \in \Gamma : \frac{dL}{dt} = 0\} \qquad (22)$$

is the endemic equilibrium of the model under consideration, located at E_p^*. It follows that E_{h*} is globally asymptotically stable in Γ if $\Sigma < \Omega$, thanks to Lasalle's invariance idea.

3. Numerical Scheme

In this section, we present a numerical solution to the problem that makes use of a Newton polynomial. Regarding the proposed paradigm, We employ the new differential and integral operators in this situation. Here, the traditional differential operator will be replaced by an operator that makes use of the Mittag-Leffler kernel. In addition, the version with a movable order will also be applied.

$$\begin{aligned}
{}_{O}^{FFM}D_t^{\xi,\psi}S_p(t) &= \Lambda_p-(\mu_p+\lambda_p)S_p(t),\\
{}_{O}^{FFM}D_t^{\xi,\psi}E_p(t) &= \lambda_p S_p(t)-(\mu_p+\omega)E_p(t),\\
{}_{O}^{FFM}D_t^{\xi,\psi}I_p(t) &= \omega E_p(t)-(\mu_p+\gamma)I_p(t),\\
{}_{O}^{FFM}D_t^{\xi,\psi}R_p(t) &= \gamma(1-f)I_p(t)-\mu_p R_p(t),\\
{}_{O}^{FFM}D_t^{\xi,\psi}S_a(t) &= \Lambda_a-(\mu_a+\lambda_a)S_a(t),\\
{}_{O}^{FFM}D_t^{\xi,\psi}I_a(t) &= \lambda_a S_a(t)-(\mu_a+\rho)I_a(t),\\
{}_{O}^{FFM}D_t^{\xi,\psi}R_a(t) &= \rho I_a(t)-\mu_a R_a(t),
\end{aligned} \tag{23}$$

To make things easier, we express the equation like this:

$$\begin{aligned}
{}_{O}^{FFM}D_t^{\xi,\psi}S_p(t) &= S_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
{}_{O}^{FFM}D_t^{\xi,\psi}E_p(t) &= E_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
{}_{O}^{FFM}D_t^{\xi,\psi}I_p(t) &= I_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
{}_{O}^{FFM}D_t^{\xi,\psi}R_p(t) &= R_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
{}_{O}^{FFM}D_t^{\xi,\psi}S_a(t) &= S_{a1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
{}_{O}^{FFM}D_t^{\xi,\psi}I_a(t) &= I_{a1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),\\
{}_{O}^{FFM}D_t^{\xi,\psi}R_a(t) &= R_{a1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a),
\end{aligned} \tag{24}$$

Where

$$\begin{aligned}
S_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a) &= \Lambda_p-(\mu_p+\lambda_p)S_p(t),\\
E_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a) &= \lambda_p S_p(t)-(\mu_p+\omega)E_p(t),\\
I_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a) &= \omega E_p(t)-(\mu_p+\gamma)I_p(t),\\
R_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a) &= \gamma(1-f)I_p(t)-\mu_p R_p(t),\\
S_{a1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a) &= \Lambda_a-(\mu_a+\lambda_a)S_a(t),\\
I_{a1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a) &= \lambda_a S_a(t)-(\mu_a+\rho)I_a(t),\\
R_{a1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a) &= \rho I_a(t)-\mu_a R_a(t),
\end{aligned} \tag{25}$$

Following application of the Mittag-Leffler kernel to the fractal fractional integral, we derive the following

$$\begin{aligned}
S_p(t_\sigma+1) &= \frac{\psi(1-\xi)}{AB(\xi)}t_\sigma^{\psi-1}S_{p1}(t_\sigma,S_p(t_\sigma),E_p(t_\sigma),I_p(t_\sigma),R_p(t_\sigma),S_a(t_\sigma),I_a(t_\sigma),R_a(t_\sigma))\\
&\quad+\frac{\psi\xi}{AB(\xi)\Gamma(\xi)}\sum_{f=2}^{\sigma}\int_{t_f}^{t_{f+1}}S_{p1}(t,S_p,E_p,I_p,R_p,S_a,I_a,R_a)\tau^{\xi-1}(t_{\sigma+1}-\tau)^{\xi-1}d\tau
\end{aligned} \tag{26}$$

$$
\begin{aligned}
E_p(t_\sigma+1) &= \frac{\psi(1-\xi)}{AB(\xi)} t_\sigma^{\psi-1} E_{p1}(t_\sigma, S_p(t_\sigma), E_p(t_\sigma), I_p(t_\sigma), R_p(t_\sigma), S_a(t_\sigma), I_a(t_\sigma), R_a(t_\sigma)) \\
&+ \frac{\psi\xi}{AB(\xi)\Gamma(\xi)} \sum_{f=2}^{\sigma} \int_{t_f}^{t_{f+1}} E_{p1}(t, S_p, E_p, I_p, R_p, S_a, I_a, R_a) \tau^{\xi-1} (t_{\sigma+1}-\tau)^{\xi-1} d\tau
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
I_p(t_\sigma+1) &= \frac{\psi(1-\xi)}{AB(\xi)} t_\sigma^{\psi-1} I_{p1}(t_\sigma, S_p(t_\sigma), E_p(t_\sigma), I_p(t_\sigma), R_p(t_\sigma), S_a(t_\sigma), I_a(t_\sigma), R_a(t_\sigma)) \\
&+ \frac{\psi\xi}{AB(\xi)\Gamma(\xi)} \sum_{f=2}^{\sigma} \int_{t_f}^{t_{f+1}} I_{p1}(t, S_p, E_p, I_p, R_p, S_a, I_a, R_a) \tau^{\xi-1} (t_{\sigma+1}-\tau)^{\xi-1} d\tau
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
R_p(t_\sigma+1) &= \frac{\psi(1-\xi)}{AB(\xi)} t_\sigma^{\psi-1} R_{p1}(t_\sigma, S_p(t_\sigma), E_p(t_\sigma), I_p(t_\sigma), R_p(t_\sigma), S_a(t_\sigma), I_a(t_\sigma), R_a(t_\sigma)) \\
&+ \frac{\psi\xi}{AB(\xi)\Gamma(\xi)} \sum_{f=2}^{\sigma} \int_{t_f}^{t_{f+1}} R_{p1}(t, S_p, E_p, I_p, R_p, S_a, I_a, R_a) \tau^{\xi-1} (t_{\sigma+1}-\tau)^{\xi-1} d\tau
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
S_a(t_\sigma+1) &= \frac{\psi(1-\xi)}{AB(\xi)} t_\sigma^{\psi-1} S_{a1}(t_\sigma, S_p(t_\sigma), E_p(t_\sigma), I_p(t_\sigma), R_p(t_\sigma), S_a(t_\sigma), I_a(t_\sigma), R_a(t_\sigma)) \\
&+ \frac{\psi\xi}{AB(\xi)\Gamma(\xi)} \sum_{f=2}^{\sigma} \int_{t_f}^{t_{f+1}} S_{a1}(t, S_p, E_p, I_p, R_p, S_a, I_a, R_a) \tau^{\xi-1} (t_{\sigma+1}-\tau)^{\xi-1} d\tau
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
I_a(t_\sigma+1) &= \frac{\psi(1-\xi)}{AB(\xi)} t_\sigma^{\psi-1} I_{a1}(t_\sigma, S_p(t_\sigma), E_p(t_\sigma), I_p(t_\sigma), R_p(t_\sigma), S_a(t_\sigma), I_a(t_\sigma), R_a(t_\sigma)) \\
&+ \frac{\psi\xi}{AB(\xi)\Gamma(\xi)} \sum_{f=2}^{\sigma} \int_{t_f}^{t_{f+1}} I_{a1}(t, S_p, E_p, I_p, R_p, S_a, I_a, R_a) \tau^{\xi-1} (t_{\sigma+1}-\tau)^{\xi-1} d\tau
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
R_a(t_\sigma+1) &= \frac{\psi(1-\xi)}{AB(\xi)} t_\sigma^{\psi-1} R_{a1}(t_\sigma, S_p(t_\sigma), E_p(t_\sigma), I_p(t_\sigma), R_p(t_\sigma), S_a(t_\sigma), I_a(t_\sigma), R_a(t_\sigma)) \\
&+ \frac{\psi\xi}{AB(\xi)\Gamma(\xi)} \sum_{f=2}^{\sigma} \int_{t_f}^{t_{f+1}} R_{a1}(t, S_p, E_p, I_p, R_p, S_a, I_a, R_a) \tau^{\xi-1} (t_{\sigma+1}-\tau)^{\xi-1} d\tau
\end{aligned} \tag{32}
$$

As we will remember, the Newton polynomial's source is

$$
\begin{aligned}
G(t, S_p, E_p, I_p, R_p, S_a, I_a, R_a) &\simeq G(t_{\sigma-2}, S_{p\sigma-2}, E_{p\sigma-2}, I_{p\sigma-2}, R_{p\sigma-2}, S_{a\sigma-2}, I_{a\sigma-2}, R_{a\sigma-2}) \\
&+ \frac{1}{\Delta t}[(t_{\sigma-2}, S_{p\sigma-2}, E_{p\sigma-1}, I_{p\sigma-1}, R_{p\sigma-1}, S_{a\sigma-1}, I_{a\sigma-1}, R_{a\sigma-1}) \\
&- G(t_{\sigma-2}, S_{p\sigma-2}, E_{p\sigma-2}, I_{p\sigma-2}, R_{p\sigma-2}, S_{a\sigma-2}, I_{a\sigma-2}, R_{a\sigma-2})] \\
&\times (\tau - t_{\sigma-2}) + \frac{1}{2\Delta t^2}[G(t_\sigma, S_{p\sigma}, E_{p\sigma}, I_{p\sigma}, R_{p\sigma}, S_{a\sigma}, I_{a\sigma}, R_{a\sigma}) \\
&- 2G[(t_{\sigma-2}, S_{p\sigma-2}, E_{p\sigma-1}, I_{p\sigma-1}, R_{p\sigma-1}, S_{a\sigma-1}, I_{a\sigma-1}, R_{a\sigma-1}) \\
&- G(t_{\sigma-2}, S_{p\sigma-2}, E_{p\sigma-2}, I_{p\sigma-2}, R_{p\sigma-2}, S_{a\sigma-2}, I_{a\sigma-2}, R_{a\sigma-2})] \\
&\times (\tau - t_{\sigma-2})(\tau - t_{\sigma-1})
\end{aligned} \tag{33}
$$

Equations should use the Newton polynomial instead(3.34-3.40),we get the following

$$\begin{aligned}
S_p^{(\sigma+1)} &= \frac{\psi(1-\xi)}{AB(\xi)}t_\sigma^{\psi-1}S_{p1}(t_\sigma,S_p(t_\sigma),E_p(t_\sigma),I_p(t_\sigma),R_p(t_\sigma),S_a(t_\sigma),I_a(t_\sigma),R_a(t_\sigma))\\
&+\frac{\psi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{f=2}^{\sigma}S_{p1}(t_{f-2},S_p^{f-2},E_p^{f-2},I_p^{f-2},R_p^{f-2},S_a^{f-2},I_a^{f-2},R_a^{f-2})t_{f-2}^{\psi-1}\\
&\times\int_{t_f}^{t_{f+1}}(t_{\sigma+1}-\tau)^{\xi-1}d\tau\\
&+\frac{\psi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{f=2}^{\sigma}\frac{1}{\Delta t}[t_{1-f}^{\psi-1}S_{p1}(t_{f-1}S_p^{f-1},E_p^{f-1},I_p^{f-1},R_p^{f-1},S_a^{f-1},I_a^{f-1},R_a^{f-1})\\
&-t_{f-2}^{\psi-1}S_{p1}(t_{f-2},S_p^{f-2},E_p^{f-2},I_p^{f-2},R_p^{f-2},S_a^{f-2},I_a^{f-2},R_a^{f-2})]\\
&\times\int_{t_f}^{t_{f+1}}(\tau-t_{f-2})(t_{\sigma+1}-\tau)^{\xi-1}d\tau\\
&+\frac{\psi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{f=2}^{\sigma}\frac{1}{2\Delta t^2}[t_f^{\psi-1}S_{p1}(t_fS_p^f,E_p^f,I_p^f,R_p^f,S_a^f,I_a^f,R_a^f)\\
&-2t_{f-1}^{\psi-1}S_{p1}(t_{f-1},S_p^{f-1},E_p^{f-1},I_p^{f-1},R_p^{f-1},S_a^{f-1},I_a^{f-1},R_a^{f-1})]\\
&+t_{f-2}^{\psi-1}S_{p1}(t_{f-2},S_p^{f-2},E_p^{f-2},I_p^{f-2},R_p^{f-2},S_a^{f-2},I_a^{f-2},R_a^{f-2})]\\
&\times\int_{t_f}^{t_{f+1}}(\tau-t_{f-2})(\tau-t_{f-1})(t_{\sigma+1}-\tau)^{\xi-1}d\tau
\end{aligned}\tag{34}$$

The following approaches can be used to determine the integral in the given equation.

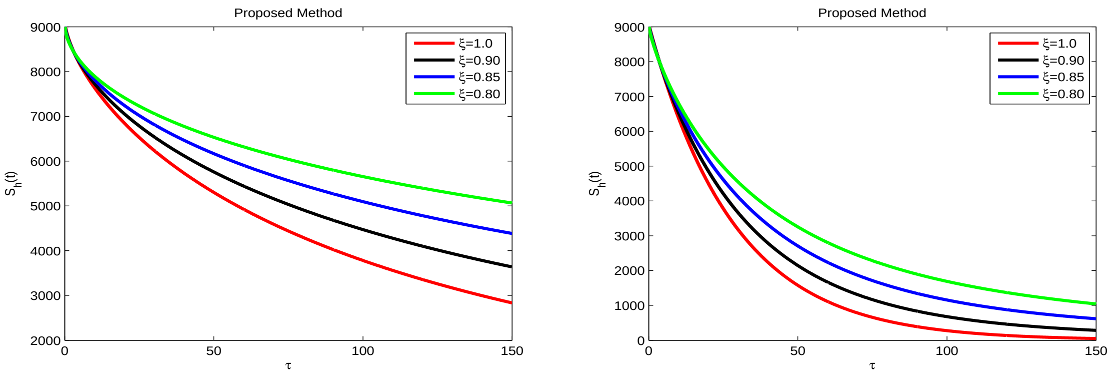
$$\begin{aligned}
\int_{t_f}^{t_{f+1}}(t_{\sigma+1}-\tau)^{\xi-1}d\tau &= \frac{(\Delta t)^\xi}{\xi}[(\sigma-f+1)^\xi-(\sigma-f)^\xi]\\
\int_{t_f}^{t_{f+1}}(\tau-t_{f-2})(t_{\sigma+1}-\tau)^{\xi-1}d\tau &= \frac{(\Delta t)^{\xi+1}}{\xi(\xi+1)}[(\sigma-f+1)^\xi\\
&\times(\sigma-f+3+2\xi)-(\sigma-f)^\xi(\sigma-f+3+3\xi)]
\end{aligned}\tag{35}$$

$$
\begin{aligned}
S_p^{(\sigma+1)} \quad = \quad & \frac{\psi(1-\xi)}{AB(\xi)} t_\sigma^{\psi-1} S_{p1}(t_\sigma, S_p(t_\sigma), E_p(t_\sigma), I_p(t_\sigma), R_p(t_\sigma), S_a(t_\sigma), I_a(t_\sigma), R_a(t_\sigma)) \\
& + \frac{\xi(\Delta t)^\xi}{AB(\xi)\Gamma(\xi+1)} \Sigma_{f=2}^{\sigma} t_{f-2}^{\psi-1} S_{p1}(t_{f-2}, S_p^{f-2}, E_p^{f-2}, I_p^{f-2}, R_p^{f-2}, S_a^{f-2}, I_a^{f-2}, R_a^{f-2}) \\
& \times[(\sigma-f+1)^\xi - (\sigma-f)^\xi] \\
& + \frac{\xi(\Delta t)^\xi}{AB(\xi)\Gamma(\xi+2)} \Sigma_{f=2}^{\sigma} [t_{1-f}^{\psi-1} S_{p1}(t_{f-1} S_p^{f-1}, E_p^{f-1}, I_p^{f-1}, R_p^{f-1}, S_a^{f-1}, I_a^{f-1}, R_a^{f-1}) \\
& - t_{f-2}^{\psi-1} S_{p1}(t_{f-2}, S_p^{f-2}, E_p^{f-2}, I_p^{f-2}, R_p^{f-2}, S_a^{f-2}, I_a^{f-2}, R_a^{f-2})] \\
& \times[(\sigma-f+1)^\xi(\sigma-f+3+2\xi) \\
& -(\sigma-f)^\xi(\sigma-f+3+3\xi)] \\
& + \frac{\xi(\Delta t)^\xi}{2AB(\xi)\Gamma(\xi+3)} \Sigma_{f=2}^{\sigma} [t_f^{\psi-1} S_{p1}(t_f S_p^f, E_p^f, I_p^f, R_p^f, S_a^f, I_a^f, R_a^f) \\
& -2t_{f-1}^{\psi-1} S_{p1}(t_{f-1}, S_p^{f-1}, E_p^{f-1}, I_p^{f-1}, R_p^{f-1}, S_a^{f-1}, I_a^{f-1}, R_a^{f-1}) \\
& + t_{f-2}^{\psi-1} S_{p1}(t_{f-2}, S_p^{f-2}, E_p^{f-2}, I_p^{f-2}, R_p^{f-2}, S_a^{f-2}, I_a^{f-2}, R_a^{f-2})] \\
& \times[(\sigma-f+1)^\xi\{2(\sigma-f)^2+(3\xi+10)(\sigma-f)+2\xi^2+9\xi+12\} \\
& -(\sigma-f)^\xi\{2(\sigma-f)^2+(5\xi+10)(\sigma-f)+6\xi^2+18\xi+12\}], \qquad (36)
\end{aligned}
$$

and similar for the all remaining equations of the model.

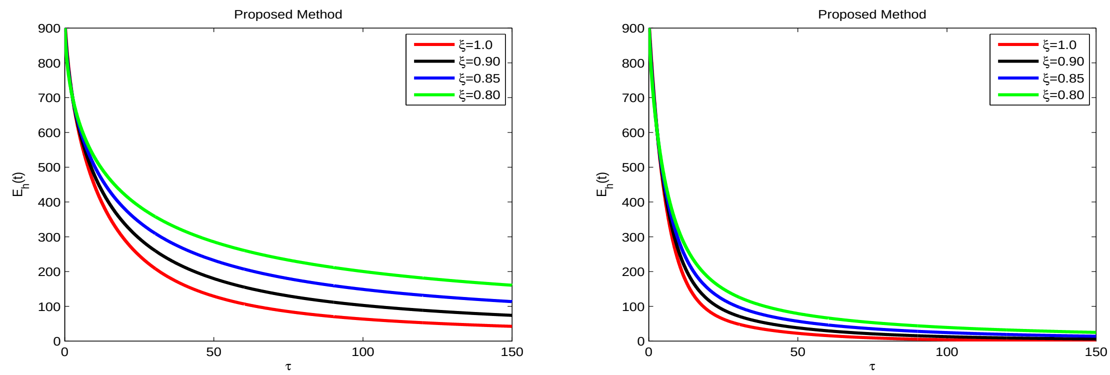
4. Results and Discussion

The following examples demonstrate the efficacy of the obtained theoretical consequences. A mathematical analysis of marburg disease is presented, and by using non-integer parametric parameters, compelling results are obtained. By reducing the fractional values and the dimension, the answer for S_p, E_p, I_p, R_p, S_a, I_a and R_a in Figure 1-7 approaches the desired value. The numerical simulation for the fractional order marburg disease model is found using Matlab code. We show the graphical representation of the marburg disease model using the suggested numerical method in Figures 1-7, and we compare the integer order result with the fractal-fractional order result. The dynamics of marburg susceptible S_p, marburg exposed E_p, infected marburg I_p, bacterial susceptible S_a and infected becterial I_a are shown in Figures 1, 2, 3, 5, and 6, respectively. In these scenarios, all of the compartments had a sloped downward inclination, and after some time, they approached a steady position because to a rise in recovered. The dynamics of marburg recover R_p and recovered becterial R_a are shown in Figures 4 and 7, respectively. In these scenarios, all of the compartments rigidly sloped upward, as the recovered situation increased, the compartments approached a stable state. Figures 1a-7a and 1b-7b, respectively, show that while behaviors are comparable when using dimensions of 1 and 0.7 with small impacts, decreasing dimensions yields more acceptable results. Additionally, as shown in Figures 4a and 4b, respectively, recovered human with and without medicine grow by decreasing the fractional values and dimension. It makes predictions on what this research will lead to in the future and how we will be able to lower the number of sick human and infected vectors that spread throughout the environment. When compared to traditional derivatives, the FFM technique yields better results for all sub-compartments at fractional derivatives. It is also suggested that as fractional values and dimensions are reduced, the solutions for all compartments become more reliable and accurate.



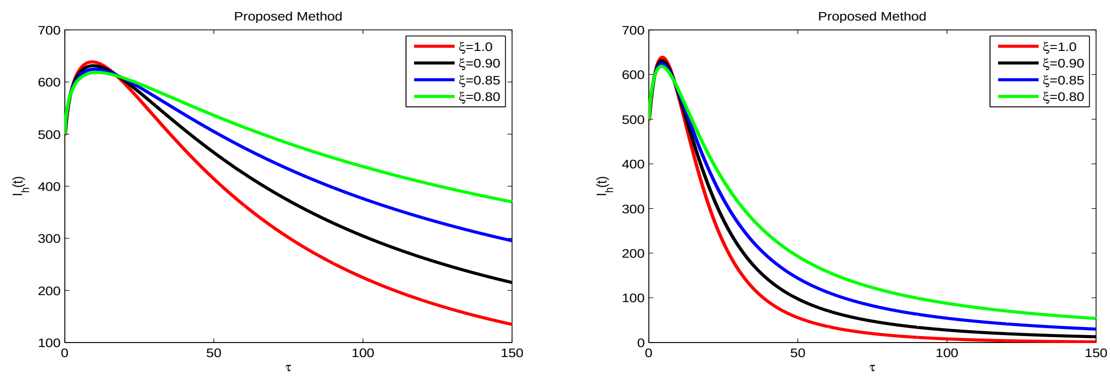
(a) Dimension = 0.7 **(b)** Dimension = 1

Figure 1: Numerical solutions for the recuperated $S_p(t)$ population.



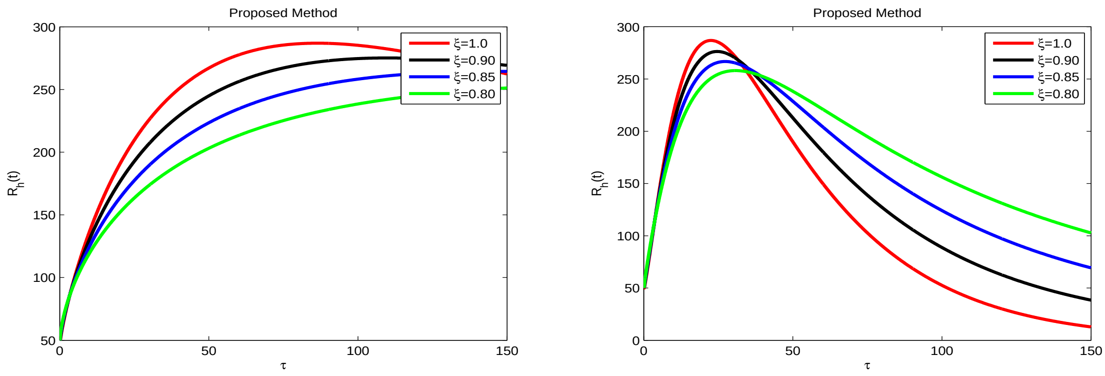
(a) Dimension = 0.7 **(b)** Dimension = 1

Figure 2: Numerical solutions for the recuperated $E_p(t)$ population.



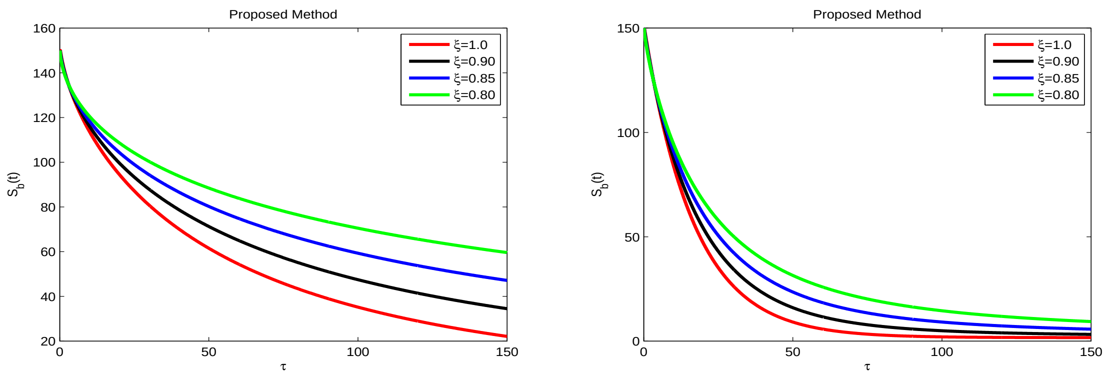
(a) Dimension = 0.7 **(b)** Dimension = 1

Figure 3: Numerical solutions for the recuperated $I_p(t)$ population.



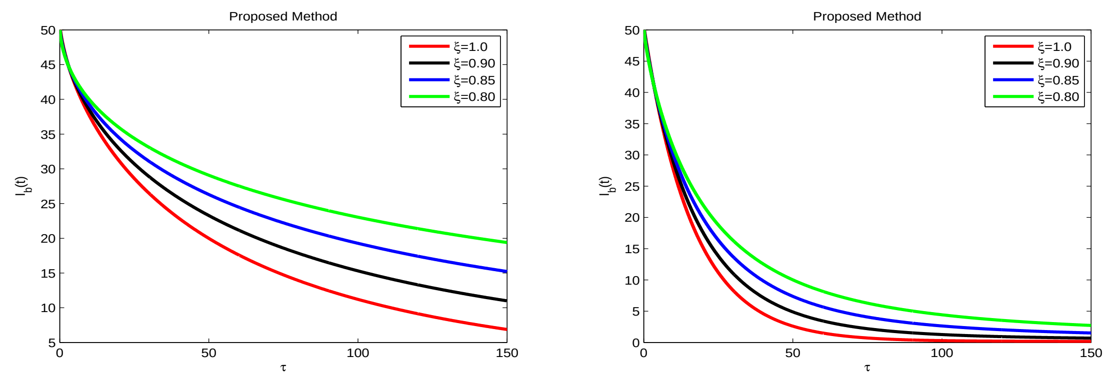
(a) Dimension = 0.7 **(b)** Dimension = 1

Figure 4: Numerical solutions for the recuperated $R_p(t)$ population.



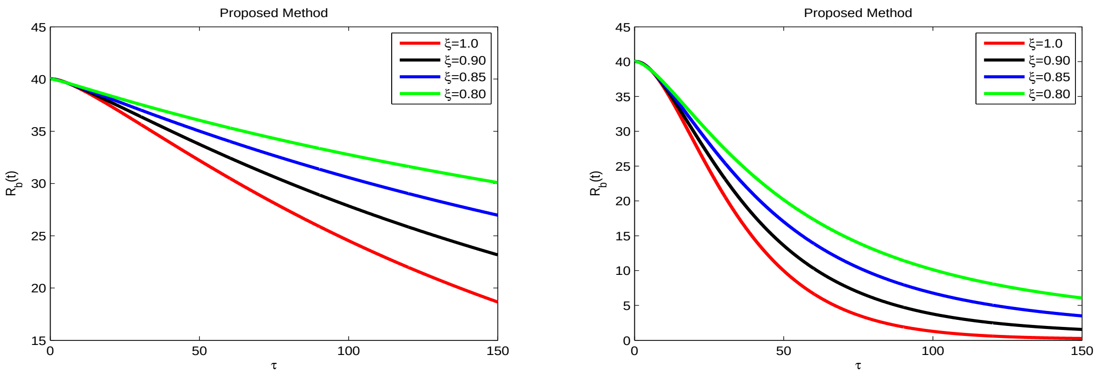
(a) Dimension = 0.7 **(b)** Dimension = 1

Figure 5: Numerical solutions for the recuperated $S_a(t)$ population.



(a) Dimension = 0.7 **(b)** Dimension = 1

Figure 6: Numerical solutions for the recuperated $I_a(t)$ population.



(a) Dimension = 0.7 **(b)** Dimension = 1

Figure 7: Numerical solutions for the recuperated $R_a(t)$ population.

5. Conclusion

It is essential to increase awareness among people during any pandemic for reducing the chance of infectious diseases spreading to individuals.Mathematical modeling serves the primary purposes of planning, controlling, and reducing the harmful consequences of infectious diseases that have historically decayed on the planet.As the fractional order model's findings on the epidemic model are more trustworthy compared to those of the traditional model. We cover the whole scenario of disease transmission using fractional derivative. In this study, we introduced the Mittag-Leffler kernel-based fractal-fractional derivative model of the epidemic Marburg virus. It has been established that the solutions are bounded and positive. Our mathematical model for the human species is of fractional order, wherein the exposed (E_p),partition joins the conventionally susceptible population (S_p), infected persons (I_p), and recovered human compartments (R_p).Similarly, the population of nonhuman animals is divided into three groups: Infected (I_a), Sensitive (S_a), and the presence and uniqueness of the solution have been thoroughly investigated. Following the determination of the reproduction number and equilibrium points, we go over the suggested model's local stability. The global stability analysis has been discussed using the Lyapunov direct approach. Sensitivity analysis is explained as a way to determine the effects ofincluded parameters in the model that was suggested.Using Newton's polynomial, We use the Mittag-Leffler kernel to calculate the numerical solutions for the proposed model. Numerical simulations for various fractal values. Fractional order (ψ) and dimension (ξ) have been built. A comparison between Orders in fractions and integers has been conducted in order to observe the effects of illness on humans and non-humans. The findings of our study will be beneficial to scientists in their further endeavors. It is important to note that the memory elements in the FF derivativewhich cannot be realized with integer-order derivativesexplore the hidden infected dynamics in mathematical models of viral diseases.Ultimately, graphical representation of all the theoretical discoveries is provided by Matlab program. Though fractional operators provide us with continuous surveillance of infectious diseases, many physical processes are theoretically expressed such that the classical derivative only reveals the dynamics in one direction and fractional orders investigate illnesses as soon as they arise. Modeling aids in the analysis of the full course of infectious behavior and the effects of control measures.Compared to previous fractional order derivatives that preserve the properties of non-singular kernels, a more practical method is available using the fractional derivative with Mittag-Leffler kernel.The fractional derivative of this The memory effect should be obtained for the models that are being given. In our work, we thereby make advantage of this new derivative. When precise Real-time transmission structure calculations are provided, and this model gains a high degree of dependability. The newly proposed adjustment will provide some light on modeling challenges both with and without singularity at the origin.

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