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Investigation of COVID-19 outbreak as a case study in Italy using different fractional operators

Aqeel Ahmad[a,*]

[a] Department of Mathematics, Ghazi University D G Khan 32200, Pakistan
[b] Mathematics Research Center, Near East University, 99138, Turkey
Corresponding author Email: aqeelahmad.740@gmail.com

Abstract: This work analyzes the COVID-19 fractional order SEIQRD compartmental model using the six primary categories of the Caputo approach. Numerous conclusions have been drawn on the the solution's boundedness and non-negativity, as well as the existence and uniqueness of the new mod el. Our findings reveal that the When $R_0 < 1$, the system is locally asymptotically stable at infection-free equilibrium. We also observed that In the absence of illness, the system is globally asymptotically stable (R of Covid 19 <1). This study's primary goal is to look into the dynamics of COVID-19 transmission in Italy, the nation where the virus was initially discovered in January 2020. To account for the uncertainty arising from the limited knowledge about the Corona virus (COVID-19), The order of fractions A fractional order was used to apply the SEIQRD compartmental model framework. The dynamics of the equilibrium are examined using the La-Salle invariant principle and the Routh-Hurwitz consistency criterion. Additionally, the suggested model's approximate solution is calculated using the fractional-order Taylor's method. Through the comparison of simulation results with empirical data, the model's validity is shown. The study's findings regarding the impacts of using face masks showed that wearing face masks frequently can prevent the COVID-19 virus from spreading.
Keywords: Mathematical modeling; Boundedness; Existence; Stability analysis;

1. Introduction

The coronavirus, or COVID-19, is a relatively new problem that has put the entire planet in danger. According to reports, its originated from Wuhan city of China [1]. In December 2019, reports of a novel coronavirus were made for the first time. Dry cough, fever, exhaustion, and severe cases of acute respiratory syndrome are among the symptoms of coronavirus, which manifest in two to ten days and can progress to pneumonia, kidney failure, and even death [2]. BThe world faced an emergency as the coronavirus spread over the world between March and April. The wet seafood market in Wuhan, China, was the site of the first documented cases [3]. The Covid-19 pandemic's fast and ongoing global spread continues to pose a serious challenge to all nations. As per the World Health Organization's (WHO) situation report-205, published on August 12, 2020, there are over 20 million COVID-19-positive cases worldwide and over 730,000 fatalities [4]. A novel coronavirus known as The sickness is caused by the coronavirus disease (COVID-19) or severe acute respiratory syndrome coronavirus 2 (SARSCoV-2). It is believed to have started in Wuhan, China, where it was initially discovered in December 2019 [5]. As we can see, the vaccine was created in China, the United States, England, and Russia. The majority of individuals on the planet are getting vaccination shots. As of 18 March 2021, at 4:53 pm CET, The World Health Organization (WHO) declared that 2,674,078 deaths and 120,915,219 confirmed cases had been registered globally. A total of 364,184,603 vaccine doses had been delivered. Rich nations like the United States, the United Kingdom, Italy, Spain, and many others are severely impacted; these nations account for the majority of recorded global fatalities [7]. The dynamics and behavior of disease are understood by mathematical modeling, which is then used to develop the protocols for disease therapy. Numerous researchers created the COVID-19 models with this goal in mind [8,9,10,11]. In the examination of mathematical models, the reproductive number plays a significant role. Reproductive number provides an explanation for the COVID-19 simulation's behavior.

Numerous studies with models and estimates of the possible quantity of COVID-19 cases in a given period of time have been reported. Ivanov [12] conducted simulations to evaluate the impact of COVID-19 on worldwide supply chains. The results showed that ongoing disruptions at different levels of the facilities may play a significant role in deciding the pandemic's aftermath. A Gaussian distribution was used in the research by Li et al. [13] to look into the COVID-19 transmission in China's Hubei Province. Subsequently, the writers offered forecasts regarding potential epidemic trends in nations like Iran, South Korea, and Italy. The outcomes demonstrated that implementing Control methods would have a major impact on the pandemic's spread pace. Previous research have estimated the transmission of COVID-19 among communities using the susceptible-infected-removed (SIR) model and its larger variations, such as numerous versions of the extended-susceptible-infected-removed (eSIR) mathematical model [14]. Fanelli and Piazza's mathematical model [15] was created to analyze the coronavirus outbreak in nations including China, Italy, and France. while accounting for susceptible-infected-recovered-deaths (SIRD) shows similarities in the rates of recovery in each of these countries. However, there was a discernible difference in the rates of infection and mortality. Using models from the exponential smoothing family, Petropoulos and Makridakis [16] provided a logical method for predicting the global COVID-19 pandemic's existence. Italy has so far put in place a number of national and local lockdown regulations in an effort to stop the virus's spread. In view of the human-to-human transmission and Given the asymptomatic transmission of COVID-19, it is critical to track the growth and fall in the number of confirmed cases and make predictions about the virus's imminent spread throughout Italy [17].This study uses To simulate and predict COVID-19 in Italy, use the susceptible-exposed-infected-quarantined recovered-dead (SEIQRD) model. driven by the aforementioned concerns. The particle swarm optimization (PSO) approach has been used to identify the model's parameters because of its high efficiency, quick convergence, and simplicity of use [18].It is crucial to remember that the epidemic model's state in the classical order model is independent of its past.However, in real life, memory is essential for analyzing how an epidemic disease spreads.A power law model was discovered to describe the intervals between a patient's doctor visits[19].It is important to remember that power law results in the Caputo fractional time derivative [20].Traditional initial and boundary conditions are allowed by Caputo fractional-order derivatives when solving real-world problems. Furthermore, Caputo fractional order produces superior results than integer order because of its nonlocal behavior and capacity for change at any given instant in time [21, 22, 23, 24, 25, 26 27].

The COVID-19 pandemic was caused by the novel coronavirus SARS-CoV-2, which has had a significant effect on world health. economy, and cultures. Comprehending and simulating the mechanics of the virus's dissemination has been crucial for efficient public health measures during the crisis. In this situation, mathematical modeling has been extremely important for both anticipating and managing the virus's spread.

One of the innovative approaches in epidemiological modeling is the use of fractional calculus, which offers a more nuanced and accurate representation of the dynamics of infectious diseases. In this study, we delve into the unique mathematical framework known as the Fractional Order SEIQRD Epidemic Model designed to capture the intricate patterns of COVID-19 transmission, and apply it to a real-world case study: Italy.

Italy, being one of the early epicenters of the pandemic, experienced a unique trajectory of COVID-19 spread with varying degrees of intervention measures and healthcare system responses. The Fractional Order SEIQRD model allows us to account for the fractional order derivatives, providing a more flexible and realistic representation of the infection dynamics.

In this research, we explore the application of this advanced modeling technique to analyze and predict the COVID-19 spreading over Italy. We think about various factors such as the effectiveness of public health interventions, vaccination campaigns, and The appearance of novel variations to develop a comprehensive understanding of the pandemic's evolution in the country. By doing so, we aim to provide valuable insights into the unique challenges faced by Italy and offer data-driven recommendations for optimizing future pandemic response strategies.

Through this case study, we demonstrate the potential of fractional calculus in epidemiological modeling, not only in the context of COVID-19 but also in understanding and managing future infectious disease outbreaks. Our findings will strengthen our capacity to respond to future health emergencies and support ongoing international efforts to contain the pandemic.

2. Mathematical Model for COVID-19

The findings done in served as inspiration for the mathematical model of $COVID-19$ transmission developed in this paper. For the purposes of this investigation, the model will have six compartments . The symbol $N(t)$ represents the total population of humans to be considered, which at any one moment consists of the compartments for susceptible (S), exposed (E), infected (I), quarantines (Q), recovered (R), and deaths (D), in that order. We now formulate the $SEIQRD$ model with a $Caputo$ operator of order $0phileq1$ and a fractional order derivative. Once the basic circumstances have been finalized, the modified model can be expressed as follows: The rates of progression from E to I are represented by e, isolation from I and recovery from I by f, recovery from I by g, death from $COVID-19disease$ in I is represented by h, recovery from Q is represented by k, and death from Q is represented by l. The requirement rate into S is represented by a, while the contact rate is represented by b.

$$
\begin{aligned}
{}_0^{FFM}D_t^{\xi,\phi}S(t) &= a - b(1-\xi\beta)S(t)E(t) - cS(t),\\
{}_0^{FFM}D_t^{\xi,\phi}E(t) &= b(1-\xi\beta)S(t)E(t) - (e+c)E(t),\\
{}_0^{FFM}D_t^{\xi,\phi}I(t) &= eE(t) - (f+g+h+c)I(t),\\
{}_0^{FFM}D_t^{\xi,\phi}Q(t) &= fI(t) - (k+l+c)Q(t),\\
{}_0^{FFM}D_t^{\xi,\phi}R(t) &= gI(t) + kQ(t) - cR(t),\\
{}_0^{FFM}D_t^{\xi,\phi}D(t) &= hI(t) + lQ(t) - cD(t).
\end{aligned} \tag{1}
$$

initial conditions,

$$S(0) > 0, E(0) > 0, I(0) > 0, Q(0) > 0, R(0) > 0, D(0) > 0. \tag{2}$$

3. Effect of global derivative

It has long been known that the most common integral in the literature is the $Riemann-Stieltjes$ integral, of which the classical integral is a specific instance.If

If classical differentiation is possible for both functions, then,

$$D_w z(x) = \frac{z'(x)}{w'(x)} \tag{3}$$

assuming that for each x in $D_{w'}$, $w'(x) \neq 0$.To do this, we will use the global derivative in place of the conventional derivative.

$$D_w(S) = a - b(1-\xi\beta)S(t)E(t) - cS(t),$$

$$D_w(E) = b(1-\xi\beta)S(t)E(t) - (e+c)E(t),$$

$$D_w(I) = eE(t) - (f+g+h+c)I(t),$$

$$D_w(Q) = fI(t) - (k+l+c)Q(t),$$

$$D_w(R) = gI(t) + kQ(t) - cR(t),$$

$$D_w(D) = hI(t) + lQ(t) - cD(t).$$

We'll assume that w is differentiable in order to keep things simple, therefore,

$$S' = w'[a - b(1-\xi\beta)S(t)E(t) - cS(t)] = z_1(t,S,E,I,Q,R,D),$$

$$E' = w'[b(1-\xi\beta)S(t)E(t) - (e+c)E(t)] = z_2(t,S,E,I,Q,R,D),$$

$$I' = w'[eE(t) - (f+g+h+c)I(t)] = z_3(t,S,E,I,Q,R,D),$$

$$Q' = w'[fI(t) - (k+l+c)Q(t)] = z_4(t,S,E,I,Q,R,D),$$

$$R' = w'[gI(t) + kQ(t) - cR(t)] = z_5(t,S,E,I,Q,R,D),$$

$$D' = w'[hI(t) + lQ(t) + cD(t)] = z_6(t,S,E,I,Q,R,D).$$

We must verify the next two requirements:
Initially,

$$|Z_1(t,S,E,I,Q,R,D)|^2 = |w'[a - b(1-\xi\beta)S(t)E(t) - cS(t)]|^2$$

$$= |w'[a - bSE + b\xi\beta SE - cS]|^2$$

$$= |w'[a + (-bE + b\xi\beta E - c)S]|^2$$

$$\leq |w'|^2[a^2 + (-bE + b\xi\beta E - c)^2|s|^2]|$$

$$\leq |w'|^2 a^2 + 6|w|'[(b^2|E|^2 + b^2\xi^2\beta^2|E|^2 + c^2)|s|^2]$$

$$\leq 2 \sup_{t\in D_{w'}} |w'|^2 a^2 + 6 \sup_{t\in D_{w'}} |w|'[(b^2 \sup_{t\in D_E} |E|^2 + b^2\xi^2\beta^2 \sup_{t\in D_I} |E|^2 + c^2)|s|^2]$$

$$\leq 2\|w'\|_\infty^2 a^2 + 6\|w'\|_\infty^2[(b^2\|E\|_\infty^2 + b^2\xi^2\beta^2\|E\|_\infty^2 + c^2)|S|^2]$$

$$\leq 2\|w'\|_\infty^2 a^2[1 + 3\frac{[b^2\|E\|_\infty^2 b^2\xi^2\beta^2\|E\|_\infty^2 + c^2]|S|^2}{a^2}]$$

$$\leq K_1(1+|S|^2)$$

So, we have:

$$3\frac{[b^2\|E\|_\infty^2 b^2\xi^2\beta^2\|E\|_\infty^2+c^2]}{a^2}<1$$

where

$$K_1=2\|w'\|_\infty^2 a^2$$

$$|Z_2(lt,S,E,I,Q,R,D)|^2=|w'[b(1-\xi\beta)S(t)E(t)-(e+c)E(t)]|^2$$

$$=|w'[b(1-\xi\beta)SE-(e+c)E]|^2$$

$$\leq 2|w'|^2[|b(1-\xi\beta)SE|^2+|-(e+c)E|^2]$$

$$\leq 2\sup_{t\in D_{w'}}[(b^2(1-\xi\beta)^2\sup_{t\in D_S}|S|^2\sup_{t\in D_E}|E|^2)+(e+c)^2|E|^2]$$

$$\leq 2\|w'\|_\infty^2[b^2(1-\xi\beta)^2\|S\|_\infty^2\|E\|_\infty^2+(e+c)^2|E|^2]$$

$$\leq 2\|w'\|_\infty^2 b^2(1-\xi\beta)^2\|S\|_\infty^2\|E\|_\infty^2[1+\frac{(e+c)^2}{b^2(1-\xi\beta)^2\|S\|_\infty^2\|I\|_\infty^2}|E|^2]$$

$$\leq K_2(1+|E|^2)$$

under the condition

$$\frac{(e+c)^2}{b^2(1-\xi\beta)^2\|S\|_\infty^2\|E\|_\infty^2}<l$$

where

$$K_2=2\|w'\|_\infty^2 b^2(1-\xi\beta)^2\|S\|_\infty^2\|E\|_\infty^2$$

$$|Z_3(t,S,E,I,Q,R,D)|^2=|w'[eE(t)-(f+g+h+c)I(t)]|^2$$

$$\leq 2|w'|^2[|eE|^2+|-(f+g+h+c)I|^2]$$

$$\leq\sup_{t\in D_{w'}}|w'|^2[e^2\sup_{t\in D_E}|E|^2+(f+g+h+c)^2|I|^2]$$

$$\leq 2\|w'\|_\infty^2 e^2\|E\|_\infty^2[1+\frac{(f+g+h+c)^2}{e^2\|E\|_\infty^2}|I|^2]$$

$$\leq K_3(1+|I|^2)$$

under the condition

$$\frac{(f+g+h+c)^2}{e^2\|E\|_\infty^2} < 1$$

where

$$K_3 = 2\|w'\|_\infty^2 e^2 \|E\|_\infty^2$$

Similarly we can find for $|Z_4(t,S,E,I,Q,R,D)|^2$, $|Z_5(t,S,E,I,Q,R,D)|^2$ and $|Z_6(t,S,E,I,Q,R,D)|^2$ Consequently, the need for linear growth is met, If

$$|Z_1(t,S_1,E,I,Q,R,D) - Z_1(t,S_2,E,I,Q,R,D|^2 = |w'(-bE + b\xi\beta E - c)(S_1 - S_2)|^2$$

$$\leq \sup_{t\in D_{w'}} |w'|^2 (3b^2 \sup_{t\in D_E} |E|^2 + 3b^2\xi^2\beta^2 \sup_{t\in D_E} |E|^2 + 3c^2) \sup_{t\in D_S} |(S_1 - S_2)|^2$$

$$\leq \|w'\|_\infty^2 (3b^2\|E\|_\infty^2 + 3b^2\xi^2\beta^2\|E\|_\infty^2 + 3c^2)\|(S_1 - S_2)\|_\infty^2$$

$$\leq \overline{K_1}\|(S_1 - S_2)\|_\infty^2$$

where

$$\overline{K_1} = \|w'\|_\infty^2 (3b^2\|E\|_\infty^2 + 3b^2\xi^2\beta^2\|E\|_\infty^2 + 3c^2)$$

If

$$|Z_2(t,S,E_1,I,Q,R,D) - Z_2(t,S,E_2,I,Q,R,D|^2 = |w'(-(e+c))(E_1 - E_2)|^2$$

$$\leq |w'|^2 (2e^2 + 2c^2)|E_1 - E_2|^2$$

$$\sup_{t\in D_E} |Z_2(t,S,E_1,I,Q,R,D) - Z_2(t,S,E_2,I,Q,R,D|^2$$

$$\leq \sup_{t\in D_{w'}} |w'|^2 (2e^2 + 2c^2) \sup_{t\in D_E} |E_1 - E_2|^2$$

$$\leq \|w'\|_\infty^2 (2e^2 + 2c^2)\|E_1 - E_2\|_\infty^2$$

$$\leq \overline{K_2}\|(E_1 - E_2)\|_\infty^2$$

where

$$\overline{K_2} = \|w'\|_\infty^2 (2e^2 + 2c^2)$$

If

$$|Z_3(t,S,E,I_1,Q,R,D) - Z_3(t,S,E,I_2,Q,R,D|^2 = |w'(-(f+g+h+c))(I_1 - I_2)|^2$$

$$\leq \sup_{t\in D_{w'}} |w'|(4(f^2 + g^2 + h^2 + c^2)) \sup_{t\in D_I} |(I_1 - I_2)|^2$$

$$\leq \|w'\|_{\infty}^{2}(4(f^{2}+g^{2}+h^{2}+c^{2}))|(I_{1}-I_{2})|_{\infty}^{2}$$

$$\leq \overline{K_{3}}\|(I_{1}-I_{2})\|_{\infty}^{2}$$

where

$$\overline{K_{3}} = \|w'\|_{\infty}^{2}4(f^{2}+g^{2}+h^{2}+c^{2})$$

If

$$|Z_{4}(t,S,E,I,Q_{1},R,D)-Z_{4}(t,S,E,I,Q_{2},R,D|^{2} = |w'(-(k+l+c))(Q_{1}-Q_{2})|^{2}$$

$$\leq \sup_{t\in D_{w'}}|w'|(3(k^{2}+l^{2}+c^{2}))\sup_{t\in D_{Q}}|(Q_{1}-Q_{2})|^{2}$$

$$\leq \|w'\|_{\infty}^{2}(3(k^{2}+l^{2}+c^{2}))\|(Q_{1}-Q_{2})\|_{\infty}^{2}$$

$$\leq \overline{K_{4}}\|(Q_{1}-Q_{2})\|_{\infty}^{2}$$

where

$$\overline{K_{4}}\|w'\|_{\infty}^{2}(3(k^{2}+l^{2}+c^{2}))$$

If

$$|Z_{5}(t,S,E,I,Q,R_{1},D)-Z_{5}(t,S,E,I,Q,R_{2},D|^{2} = |w'(-(c))(R_{1}-R_{2})|^{2}$$

$$\leq \sup_{t\in D_{w'}}|w'|^{2}(c^{2}+\varepsilon_{1})\sup_{t\in D_{R}}|(R_{1}-R_{2})|^{2}$$

$$\|Z_{5}(t,S,E,I,Q,R_{1},D)-Z_{5}(t,S,E,I,Q,R_{2},D\|_{\infty}^{2}$$

$$\leq \|w'\|_{\infty}^{2}(c^{2}+\varepsilon_{1})\|(R_{1}-R_{2})\|_{\infty}^{2}$$

$$\leq \overline{K_{5}}\|(Q1-Q_{2})\|_{\infty}^{2}$$

where

$$\overline{K_{5}} = \|w'\|_{\infty}^{2}(c^{2}+\varepsilon_{1})$$

If

$$|Z_{6}(t,S,E,I,Q,R,D_{1})-Z_{6}(t,S,E,I,Q,R,D_{2}|^{2} = |w'(-(c))(D_{1}-D_{2})|^{2}$$

$$\leq \sup_{t\in D_{w'}}|w'|^{2}(c^{2}+\varepsilon_{1})\sup_{t\in D_{D}}|(D_{1}-D_{2})|^{2}$$

$$\|Z_{6}(t,S,E,I,Q,R,D_{1})-Z_{6}(t,S,E,I,Q,R,D_{2}\|_{\infty}^{2}$$

$$\leq \|w'\|_{\infty}^{2}(c^{2}+\varepsilon_{2})\|(D_{1}-D_{2})\|_{\infty}^{2}$$

$$\leq \overline{K_{6}}\|(D1-D_{2})\|_{\infty}^{2}$$

where

$$\overline{K_{6}} = \|w'\|_{\infty}^{2}(c^{2}+\varepsilon_{2})$$

4. Lyapunov First derivative

Here is the endemic equilibrium E^* for Lyapunov function: $\{$S,E,I,Q,R,D$\}$ L <0:

Theorem 1: The endemic points E of the *SEIQRD* model are globally asymptotically stable when $R_0 > 1$.

Proof: The following can be used to express the Lyapunov function: Consequently, if we take the derivative on both sides with regard to t and after simplification, we obtain:

$$\frac{dL}{dt} = (\frac{S-S^*}{S})(a-b(1-\xi\beta)SE-cS)$$

$$+(\frac{E-E^*}{E})(b(1-\xi\beta)SE-(e+c)S)+(\frac{I-I^*}{I})(eE-(f+g+h+c)I)$$

$$+(\frac{Q-Q^*}{Q})(fI-(k+l+c)Q)+(\frac{R-R^*}{R})(gI+kQ-cR))+(\frac{D-D^*}{D})$$

$$(hI+lQ-cD))$$

Putting$S=S-S^*, E=E-E^*, I=I-I^*, Q=Q-Q^*, R=R-R^*, D=D-D^*$ leads to

$$\frac{dL}{dt} = (\frac{S-S^*}{S})(a-b(1-\xi\beta)(S-S^*)(E-E^*)-c(S-S^*))+(\frac{E-E^*}{E})(b(1-\xi\beta)$$

$$(S-S^*)(E-E^*)-(e+c)(S-S^*))+(\frac{I-I^*}{I})(e(E-E^*)-(f+g+h+c)(I-I^*))$$

$$+(\frac{Q-Q^*}{Q})(f(I-I^*)-(k+l+c)((Q-Q^*))+(\frac{R-R^*}{R})(g(I-I^*))+k(Q-Q^*)$$

$$-c(R-R^*))+(\frac{D-D^*}{D})(h(I-I^*))+l(Q-Q^*)-c(D-D^*)) \tag{4}$$

After simplification, We have::

$$\frac{dL}{dt} = a-\frac{S^*}{S}a-\frac{(S-S^*)^2}{S}bE+\frac{(S-S^*)^2}{S}bE^*+\frac{(S-S^*)^2}{S}b\xi\beta E-\frac{(S-S^*)^2}{S}b\xi\beta E^*$$

$$-\frac{(S-S^*)^2}{S}c+bSE-\frac{E^*}{E}bS^*E^*-b\xi\beta SE-\frac{E^*}{E}b\xi\beta S^*E^*-\frac{(E-E^*)^2}{E}(e+c)+eE+$$

$$\frac{E^*}{E}eE^*-\frac{(I-I^*)^2}{I}(f+g+h+c)+fI+\frac{Q^*}{Q}fI^*-\frac{(Q-Q^*)^2}{Q}(k+l+c)+gI+\frac{R^*}{R}gI^*$$

$$+kQ+\frac{R^*}{R}kQ-\frac{(R-R^*)^2}{R}c+hI+\frac{D^*}{D}hI^*+lQ+\frac{D^*}{D}Q^*-\frac{(D-D^*)^2}{D}c$$

So we may have:

$$\frac{dL}{dt} = (\Sigma)-(\Omega) \tag{5}$$

and

$$\Sigma = a + \frac{(S-S^*)^2}{S} bE^* + \frac{(S-S^*)^2}{S} b\xi\beta E + bSE + eE + \frac{I^*}{I} eE^*$$

$$+ fI + \frac{Q^*}{Q} fI^* + gI + \frac{R^*}{R} gI^* + kQ + \frac{R^*}{R} kQ + hI + \frac{D^*}{D} hI^* + lQ + \frac{D^*}{D} lQ^*$$

and

$$\Omega = \frac{S^*}{S} a + \frac{(S-S^*)^2}{S} bE + \frac{(S-S^*)^2}{S} b\xi\beta E^*$$

$$+ \frac{(S-S^*)^2}{S} c + \frac{E^*}{E} bS^*E^* + b\xi\beta SE + \frac{E^*}{E} b\xi\beta S^*E^* + \frac{(E-E^*)^2}{E}(e+c)$$

$$+ \frac{(I-I^*)^2}{I}(f+g+h+c) + \frac{(Q-Q^*)^2}{Q}(k+l+c) + \frac{(R-R^*)^2}{R} c + \frac{(D-D^*)^2}{D} c$$

It is concluded that $(\Sigma) < (\Omega)$ this yields $(\frac{dL}{dt}) < 0$; however when $S = S^*; E = E^*; I = I^*; Q = Q^*; R = R^*; D = D^*$

$$0 = (\Sigma) - (\Omega) \Rightarrow (\frac{dL}{dt}) = 0 \tag{6}$$

So,

$$\{(S^*, E^*, I^*, Q^*, R^*, D^*) \in \Gamma : \frac{dL}{dt} = 0\} \tag{7}$$

According to Lasalles invariance system is globally asymptotically stable.

5. Second derivative with Lyapunov function

It should be highlighted that while a Lyapunov function's sign can help explain its stability, a function's variabilities cannot be completely understood by examining its first derivative alone. We utilize second derivative as:

$$\frac{d\dot{L}}{dt} = \frac{d}{dt}[\{(1-\frac{S^*}{S})\dot{S}] + (1-\frac{E^*}{E})\dot{E} + (1-\frac{I^*}{I})\dot{I} + (1-\frac{Q^*}{Q})\dot{Q} +$$

$$(1-\frac{R^*}{R})\dot{R} + (1-\frac{D^*}{D})\dot{D}\}$$

$$= (\frac{\dot{S}}{S})^2 S^* + (\frac{\dot{E}}{E})^2 E^* + (\frac{\dot{I}}{I})^2 I^* + (\frac{\dot{Q}}{Q})^2 Q^* + (\frac{\dot{R}}{R})^2 R^* + (\frac{\dot{D}}{D})^2 D^* + (1-\frac{S^*}{S})\ddot{S}$$

$$+ [(1-\frac{E^*}{E})] \times \ddot{E} + [(1-\frac{I^*}{I})] \times \ddot{I} + (1-\frac{Q^*}{Q})\ddot{Q} + [(1-\frac{R^*}{R})] \times \ddot{R} + (1-\frac{D^*}{D})\ddot{D}$$

here

$$\ddot{S} = a - b(1-\xi\beta)(\dot{S}E + S\dot{E}) - c\dot{S},$$

$$\ddot{E} = b(1-\xi\beta)(\dot{S}E + S\dot{E}) - (e+c)\dot{E},$$

$$\ddot{I} = e\dot{E} - (f+g+h+c)\dot{I},$$

$$\ddot{Q} = f\dot{I} - (k+l+c)\dot{Q},$$

$$\ddot{R} = g\dot{I} + k\dot{Q} - c\dot{R},$$

$$\ddot{D} = h\dot{I} + l\dot{Q} - c\dot{D}.$$

then we have

$$\frac{d\dot{L}}{dt} = (\frac{\dot{S}}{S})^2 S^* + (\frac{\dot{E}}{E})^2 E^* + (\frac{\dot{I}}{I})^2 I^* + (\frac{\dot{Q}}{Q})^2 Q^* + (\frac{\dot{R}}{R})^2 R^* + (\frac{\dot{D}}{D})^2 D^* + (1 - \frac{S^*}{S})$$

$$(a - b(1-\xi\beta)(\dot{S}E + S\dot{E}) - c\dot{S}) + (1 - \frac{E^*}{E})(b(1-\xi\beta)(\dot{S}E + S\dot{E}) - (e+c)\dot{E})$$

$$+ (1 - \frac{I^*}{I})(e\dot{E} - (f+g+h+c)\dot{I}) + (1 - \frac{Q^*}{Q})(f\dot{I} - (k+l+c)\dot{Q}) + (1 - \frac{R^*}{R})$$

$$+ (g\dot{I} + k\dot{Q} - c\dot{R})s(1 - \frac{D^*}{D})(h\dot{I} + l\dot{Q} - c\dot{D})$$

and

$$\frac{d^2L}{dt^2} = \prod(S,E,I,Q,R,D) - a(1 - \frac{S^*}{S}) - b(1 - \frac{S^*}{S})(\dot{S}E + S\dot{E}) + b\xi\beta(1 - \frac{S^*}{S})$$

$$(\dot{S}E + S\dot{E}) - c(1 - \frac{S^*}{S})\dot{S} + b(1 - \frac{S^*}{S})(\dot{S}E + S\dot{E}) - b\xi\beta(1 - \frac{E^*}{E})(\dot{S}E + S\dot{E}) -$$

$$(e+c)(1 - \frac{E^*}{E})\dot{E} + e(1 - \frac{I^*}{I})\dot{E} - (f+g+h+c)(1 - \frac{I^*}{I})\dot{I} + f(1 - \frac{Q^*}{Q})\dot{I} - (k+$$

$$l+c)(1 - \frac{Q^*}{Q})\dot{Q} + g(1 - \frac{R^*}{R})\dot{I} + k(1 - \frac{R^*}{R})\dot{Q} - c(1 - \frac{R^*}{R})\dot{R} + h(1 - \frac{D^*}{D})\dot{I} + l(1 - \frac{D^*}{D})$$

$$\dot{Q} - c(1 - \frac{D^*}{D})\dot{D}$$

We replace $\dot{S}(t); \dot{E}(t); \dot{I}(t); \dot{Q}(t); \dot{R}(t); \dot{D}(t)$ by their derivative values, we can get:

$$\frac{d^2L}{dt^2} = \prod(S,E,I,Q,R,D) - a(1 - \frac{S^*}{S}) - b(1 - \frac{S^*}{S})((a - b(1-\xi\beta)SE - cS)I +$$

$$+ S(eE - (f+g+h+c)I)) + b\xi\beta(1 - \frac{S^*}{S})((a - b(1-\xi\beta)SE - cS)I +$$

$$+ S(eE - (f+g+h+c)I)) - c(1 - \frac{S^*}{S})(a - b(1-\xi\beta)SE - cS) + b(1 - \frac{E^*}{E})$$

$$((a-b(1-\xi\beta)SE-cS)I+S(eE-(f+g+h+c)I))-b\xi\beta(1-\frac{E^*}{E})$$

$$((a-b(1-\xi\beta)SE-cS)I+S(eE-(f+g+h+c)I))-(e+c)(1-\frac{E^*}{E})$$

$$(b(1-\xi\beta)SE-(e+c)E)-e(1-\frac{I^*}{I})(b(1-\xi\beta)SE-(e+c)E)-(f+g+h+c)$$

$$(1-\frac{I^*}{I})(eE-(f+g+h+c)I)+f(1-\frac{Q^*}{Q})(eE-(f+g+h+c)I)$$

$$-(k+l+c)(1-\frac{Q^*}{Q})(fI-(k+l+c)Q)+g(1-\frac{R^*}{R})$$

$$(eE-(f+g+h+c)I)+k(1-\frac{R^*}{R})(fI-(k+l+c)Q)-c(1-\frac{R^*}{R})$$

$$(gI+kQ-cR)+h(1-\frac{D^*}{D})(eE-(f+g+h+c)I)+l(1-\frac{D^*}{D})(fI-$$

$$(k+l+c)Q)-c(1-\frac{D^*}{D})(hI+lQ-cD) \tag{8}$$

for simplicity we can write

$$\frac{d^2L}{dt^2}=\Omega_1-\Omega_2 \tag{9}$$

$$\frac{d^2L}{dt^2}=\dot{\prod}(S,E,I,Q,R,D)-abI+b^2SI^2-b^2\xi\beta SI^2-bcSI+a\frac{S^*}{S}bI$$

$$-b^2\frac{S^*}{S}SI^2+b^2\xi\beta\frac{S^*}{S}SI^2+bc\frac{S^*}{S}SI-bcSE+b(f+g+h+c)SI+bc\frac{S^*}{S}SE$$

$$-b(f+g+h+c)\frac{S^*}{S}SI-ab\xi\beta I+b^2\xi\beta SI^2-bc\xi\beta SI-ab\xi\beta\frac{S^*}{S}I-b^2\xi\beta\frac{S^*}{S}SI^2$$

$$-b^2\xi^2\beta^2\frac{S^*}{S}SI^2+bc\xi\beta\frac{S^*}{S}SI+bc\xi\beta\frac{S^*}{S}SI-b\xi\beta(f+g+h+c)SI+bc\xi\beta\frac{S^*}{S}SI$$

$$+b\xi\beta(f+g+h+c)\frac{S^*}{S}SI-ac+bcSI-bc\xi\beta SI+c^2S+ac\frac{S^*}{S}-bc\frac{S^*}{S}SI+bc$$

$$\xi\beta\frac{S^*}{S}SI-c^2\frac{S^*}{S}S+ab-b^2SI^2+b^2\xi^2\beta^2SI^2-bcSI-ab\frac{E^*}{E}+b^2\frac{E^*}{E}SI^2-b^2\xi^2\beta^2$$

$$\frac{E^*}{E}SI^2+bc\frac{E^*}{E}SI+beSE-b(f+g+h+c)SI-be\frac{E^*}{E}SE+b(f+g+h+c)\frac{E^*}{E}SI$$

$$-ab\xi\beta I-b^2\xi\beta SI^2-b^2\xi^2\beta^2SI^2+bc\xi\beta SI+ab\xi\beta\frac{E^*}{E}I+b^2\xi\beta\frac{E^*}{E}SI^2+b^2\xi^2\beta^2$$

$$\frac{E^*}{E}SI^2-bc\xi\beta\frac{E^*}{E}SI-be\xi\beta SE+b\xi\beta(f+g+h+c)SI+be\xi\beta\frac{E^*}{E}SE-b\xi\beta(f$$

$$+g+h+c)\frac{E^*}{E}SI-b(e+c)SI+b\xi\beta(e+c)SI+(e+c)^2E+b(e+c)\frac{E^*}{E}SI+b\xi$$

$$\beta(e+c)\frac{E^*}{E}SI-(e+c)^2\frac{E^*}{E}E-beSI+be\xi\beta SI+e(e+c)E+be\frac{I^*}{I}SI$$

$$-be\xi\beta\frac{I^*}{I}SI-e(e+c)\frac{I^*}{I}E-e(f+g+h+c)E+(f+g+h+c)^2I$$

$$+e(f+g+h+c)\frac{I^*}{I}E-(f+g+h+c)^2\frac{I^*}{I}I+efE-f(f+g+h+c)I-ef\frac{Q^*}{Q}E$$

$$+f(f+g+h+c)\frac{Q^*}{Q}I-f(k+l+c)I+(k+l+c)^2Q+f(k+l+c)\frac{Q^*}{Q}I-(k$$

$$+l+c)^2\frac{Q^*}{Q}Q+egE-g(f+g+h+c)I-eg\frac{R^*}{R}E+g(f+g+h+c)\frac{R^*}{R}I+fkI$$

$$-k(k+l+c)Q-fk\frac{R^*}{R}I+k(k+l+c)\frac{R^*}{R}Q-cgI-ckQ+c^2R+cg\frac{R^*}{R}I+ck\frac{R^*}{R}Q$$

$$-c^2\frac{R^*}{R}R+ehE-h(f+g+h+c)I-eh\frac{D^*}{D}E+h(f+g+h+c)D^*DI+flI-l(k$$

$$+l+c)Q-fl\frac{D^*}{D}I+l(k+l+c)\frac{D^*}{D}Q-chI-clQ+c^2D+ch\frac{D^*}{D}I+cl\frac{D^*}{D}Q-c^2\frac{D^*}{D}D$$

where

$$\Omega_1=\dot{\prod}(S,E,I,Q,R,D)+b^2SI^2+a\frac{S^*}{S}bI+b^2\xi\beta\frac{S^*}{S}SI^2+bc\frac{S^*}{S}SI$$

$$+b(f+g+h+c)SI+bc\frac{S^*}{S}SE+b^2\xi\beta SI^2+bc\xi\beta\frac{S^*}{S}SI+b\xi\beta(f+g+h$$

$$+c)\frac{S^*}{S}SI+bcSI+c^2S+ac\frac{S^*}{S}+bc\xi\beta\frac{S^*}{S}SI+ab+b^2\xi^2\beta^2SI^2+b^2\frac{E^*}{E}SI^2$$

$$+bc\frac{E^*}{E}SI+beSE+b(f+g+h+c)\frac{E^*}{E}SI+bc\xi\beta SI+ab\xi\beta\frac{E^*}{E}I+b^2\xi\beta\frac{E^*}{E}SI^2$$

$$+b^2\xi^2\beta^2\frac{E^*}{E}SI^2+b\xi\beta(f+g+h+c)SI+be\xi\beta\frac{E^*}{E}SE+b\xi\beta(e+c)SI$$

$$+(e+c)^2E+b(e+c)\frac{E^*}{E}SI+b\xi\beta(e+c)\frac{E^*}{E}SI+be\xi\beta SI+e(e+c)E+be\frac{I^*}{I}SI$$

$$+(f+g+h+c)^2I+e(f+g+h+c)\frac{I^*}{I}E+efE+f(f+g+h+c)\frac{Q^*}{Q}I+(k+$$

$$l+c)^2Q+f(k+l+c)\frac{Q^*}{Q}I+egE+g(f+g+h+c)\frac{R^*}{R}I+fkI+k(k+l+c)$$

$$\frac{R^*}{R}Q+c^2R+cg\frac{R^*}{R}I+ck\frac{R^*}{R}Q+ehE+h(f+g+h+c)D^*DI+flI+l(k$$

$$+l+c)\frac{D^*}{D}Q+c^2D+ch\frac{D^*}{D}I+cl\frac{D^*}{D}Q \tag{10}$$

and

$$\Omega_2=abI+b^2\xi\beta SI^2+bcSI+b^2\frac{S^*}{S}SI^2+bcSE+b(f+g+h+c)\frac{S^*}{S}SI+ab\xi\beta I$$

$$+be\xi\beta SI+ab\xi\beta\frac{S^*}{S}I+b^2\xi\beta\frac{S^*}{S}SI^2+b^2\xi^2\beta^2\frac{S^*}{S}SI^2+b\xi\beta(f+g+h+c)SI+$$

$$ac+bc\xi\beta SI+bc\frac{S^*}{S}SI+c^2\frac{S^*}{S}S+b^2SI^2+bcSI+ab\frac{E^*}{E}+b^2\xi^2\beta^2\frac{E^*}{E}SI^2$$

$$+b(f+g+h+c)SI+be\frac{E^*}{E}SE+ab\xi\beta I+b^2\xi\beta SI^2+b^2\xi^2\beta^2SI^2+bc\xi\beta\frac{E^*}{E}SI$$

$$+be\xi\beta SE+b\xi\beta(f+g+h+c)\frac{E^*}{E}SI+b(e+c)SI+(e+c)^2\frac{E^*}{E}E+beSI+be$$

$$\xi\beta\frac{I^*}{I}SI+be\xi\beta\frac{I^*}{I}SI+e(e+c)\frac{I^*}{I}E+e(f+g+h+c)E+(f+g+h+c)^2\frac{I^*}{I}I+$$

$$f(f+g+h+c)I+ef\frac{Q^*}{Q}E+f(k+l+c)I+(k+l+c)^2Q+(k+l+c)^2\frac{Q^*}{Q}Q+$$

$$g(f+g+h+c)I+eg\frac{R^*}{R}E+k(k+l+c)Q+fk\frac{R^*}{R}I+cgI+ckQ+c^2\frac{R^*}{R}R+h(f+g$$

$$+h+c)I+eh\frac{D^*}{D}E+l(k+l+c)Q+fl\frac{D^*}{D}I+chI+clQ+c^2\frac{D^*}{D}D \tag{11}$$

6. Fractal fractional derivative

This section describes a numerical approach that uses a Newton polynomial to solve the model numerically. In this case, we apply the new differential and integral operators to the proposed model.Here, the usual differential operator will be replaced by an operator with a Mittag-Leffler kernal. Additionally, we have:

$$0^{FFM}D_t^{\xi,\phi}tS(t) = a - b(1-\xi\beta S(t)E(t) - cS(t),$$

$$0^{FFM}D_t^{\xi,\phi}E(t) = b(1-\xi\beta S(t)E(t) - (e+c)E(t),$$

$$0^{FFM}D_t^{\xi,\phi}I(t) = eE(t) - (f+g+h+c)I(t),$$

$$0^{FFM}D_t^{\xi,\phi}Q(t) = fI(t) - (k+l+c)Q(t),$$

$$0^{FFM}D_t^{\xi,\phi}R(t) = gI(t) + kQ(t) - cR(t),$$

$$0^{FFM}D_t^{\xi,\phi}D(t) = hI(t) + lQ(t) - cD(t).$$

For simplicity, we express the following equation as follows:

$$S_1(t,S,M) = a - b(1-\xi\beta S(t)E(t) - cS(t),$$

$$E_1(t,S,M) = b(1-\xi\beta S(t)E(t) - (e+c)E(t),$$

$$I_1(t,S,M) = eE(t) - (f+g+h+c)I(t),$$

$$Q_1(t,S,E,I,Q,R,D) = fI(t) - (k+l+c)Q(t),$$

$$R_1(t,S,M) = gI(t) + kQ(t) - cR(t),$$

$$D_1(t,S,M) = hI(t) + lQ(t) - cD(t).$$

where $M = E, I, Q, R, D$ and ollowing the use of the Mittag-Leffler kernal with the fractal-fraction integral, we have

$$S(t_\varsigma+1) = \frac{\phi(1-\xi)}{AB(\phi)}t_\varsigma^{\phi-1}S_1(t_\varsigma, S(t_\varsigma), G)\frac{\phi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{\nu=2}^{\varsigma}$$

$$\int_{t_\nu}^{t_{\nu+1}} S_1(t,S,M)\tau^{\phi-1}(t_{\varsigma+1}-\pi)^{\phi-1}d\eta$$

Where $G = E(t_\varsigma), I(t_\varsigma), Q(t_\varsigma), R(t_\varsigma), D(t_\varsigma)$ and Similarly, we may derive for $E(t_\varsigma+1), I(t_\varsigma+1), Q(t_\varsigma+1), R(t_\varsigma+1), D(t_\varsigma+1)$

Now, Newton polynomial is substituted into equations, we get results as.

$$S^{(\varsigma+1)} = \frac{\phi(1-\xi)}{AB(\phi)} t_\varsigma^{\phi-1} S_1(t_\varsigma, S(t_\varsigma), G) + \frac{\phi\xi}{AB(\xi)\Gamma(\xi)}$$

$$\Sigma_{\nu=2}^{\varsigma} S_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2}) t_{\chi-2}^{\phi-1} \times \int_{t_\chi}^{t_{\chi+1}} (t_{\varsigma+1}-\eta)^{\xi-1} d\eta +$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} \frac{1}{\varsigma t} [t_{\chi-1}^{\phi-1} S_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) - t_{\chi-2}^{\phi-1}$$

$$S_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(t_{\varsigma+1}-\eta)^{\xi-1} d\eta +$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} \frac{1}{2\varsigma t^2} [t_\chi^{\phi-1} S_1(t_\chi, S^\chi, E^\chi, I^\chi, Q^\chi, R^\chi, D^\chi) - 2t_{\chi-1}^{\phi-1}$$

$$S_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) + \int_{t_\chi}^{t_{\chi+1}} t_{\chi-2}^{\phi-1} S_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2},$$

$$I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(\eta - t_{\chi-1})(t_{\varsigma+1}-\eta)^{\xi-1} d\eta,$$

$$E^{(\varsigma+1)} = \frac{\phi(1-\xi)}{AB(\phi)} t_\varsigma^{\phi-1} E_1(t_\varsigma, S(t_\varsigma), G) + \frac{\phi\xi}{AB(\xi)\Gamma(\xi)}$$

$$\Sigma_{\chi=2}^{\varsigma} E_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2}) t_{\chi-2}^{\phi-1} \times \int_{t_\chi}^{t_{\chi+1}} (t_{\varsigma+1}-\eta)^{\xi-1} d\eta$$

$$+ \frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} + \frac{1}{\varsigma t} [t_{\chi-1}^{\phi-1} E_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) - t_{\chi-2}^{\phi-1}$$

$$E_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(t_{\varsigma+1}-\eta)^{\xi-1} d\eta +$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} \frac{1}{2\varsigma t^2} [t_\chi^{\phi-1} E_1(t_\chi, S^\chi, E^\chi, I^\chi, Q^\chi, R^\chi, D^\chi) - 2t_{\chi-1}^{\phi-1}$$

$$E_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) + \int_{t_\chi}^{t_{\chi+1}} t_{\chi-2}^{\phi-1} E_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2},$$

$$I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(\eta - t_{\chi-1})(t_{\varsigma+1}-\eta)^{\xi-1} d\eta,$$

$$I^{(\varsigma+1)} = \frac{\phi(1-\xi)}{AB(\phi)} t_\varsigma^{\phi-1} I_1(t_\varsigma, S(t_\varsigma), G) + \frac{\phi\xi}{AB(\xi)\Gamma(\xi)}$$

$$\Sigma_{\chi=2}^{\varsigma} I_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2}) t_{\chi-2}^{\phi-1} \times \int_{t_\chi}^{t_{\chi+1}} (t_{\varsigma+1} - \eta)^{\xi-1} d\eta +$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} \frac{1}{\varsigma t} [t_{\chi-1}^{\phi-1} I_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) - t_{\chi-2}^{\phi-1}$$

$$I_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(t_{\varsigma+1} - \eta)^{\xi-1} d\eta +$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} \frac{1}{2\varsigma t^2} [t_\chi^{\phi-1} I_1(t_\chi, S^\chi, E^\chi, I^\chi, Q^\chi, R^\chi, D^\chi) - 2t_{\chi-1}^{\phi-1}$$

$$I_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) + \int_{t_\chi}^{t_{\chi+1}} t_{\chi-2}^{\phi-1} I_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2},$$

$$I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(\eta - t_{\chi-1})(t_{\varsigma+1} - \eta)^{\xi-1} d\eta,$$

$$Q^{(\varsigma+1)} = \frac{\phi(1-\xi)}{AB(\phi)} t_\varsigma^{\phi-1} Q_1(t_\varsigma, S(t_\varsigma), G) + \frac{\phi\xi}{AB(\xi)\Gamma(\xi)}$$

$$\Sigma_{\chi=2}^{\varsigma} Q_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2}) t_{\chi-2}^{\phi-1} \times \int_{t_\chi}^{t_{\chi+1}} (t_{\varsigma+1} - \eta)^{\xi-1} d\eta$$

$$+ \frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} \frac{1}{\varsigma t} [t_{\chi-1}^{\phi-1} Q_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) - t_{\chi-2}^{\phi-1}$$

$$Q_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(t_{\varsigma+1} - \eta)^{\xi-1} d\eta +$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)} \Sigma_{\chi=2}^{\varsigma} \frac{1}{2\varsigma t^2} [t_\chi^{\phi-1} Q_1(t_\chi, S^\chi, E^\chi, I^\chi, Q^\chi, R^\chi, D^\chi) - 2t_{\chi-1}^{\phi-1}$$

$$Q_1(t_{\chi-1}, S^{\chi-1}, E^{\chi-1}, I^{\chi-1}, Q^{\chi-1}, R^{\chi-1}, D^{\chi-1}) + \int_{t_\chi}^{t_{\chi+1}} t_{\chi-2}^{\phi-1} S_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2},$$

$$I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2})] \times \int_{t_\chi}^{t_{\chi+1}} (\eta - t_{\chi-2})(\eta - t_{\chi-1})(t_{\varsigma+1} - \eta)^{\xi-1} d\eta,$$

$$R^{(\varsigma+1)} = \frac{\phi(1-\xi)}{AB(\phi)} t_\varsigma^{\phi-1} R_1(t_\varsigma, S(t_\varsigma), G) + \frac{\phi\xi}{AB(\xi)\Gamma(\xi)}$$

$$\Sigma_{\chi=2}^{\varsigma} R_1(t_{\chi-2}, S^{\chi-2}, E^{\chi-2}, I^{\chi-2}, Q^{\chi-2}, R^{\chi-2}, D^{\chi-2}) t_{\chi-2}^{\phi-1} \times \int_{t_\chi}^{t_{\chi+1}} (t_{\varsigma+1} - \eta)^{\xi-1} d\eta +$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{\chi=2}^{\varsigma}\frac{1}{\varsigma t}[t_{\chi-1}^{\phi-1}R_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})-t_{\chi-2}^{\phi-1}$$

$$R_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]\times\int_{t_\chi}^{t_{\chi+1}}(\eta-t_{\chi-2})(t_{\varsigma+1}-\eta)^{\xi-1}d\eta+$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{\chi=2}^{\varsigma}\frac{1}{2\varsigma t^2}[t_{\chi}^{\phi-1}R_1(t_\chi,S^\chi,E^\chi,I^\chi,Q^\chi,R^\chi,D^\chi)-2t_{\chi-1}^{\phi-1}$$

$$R_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})+\int_{t_\chi}^{t_{\chi+1}}t_{\chi-2}^{\phi-1}R_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},$$

$$I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]\times\int_{t_\chi}^{t_{\chi+1}}(\eta-t_{\chi-2})(\eta-t_{\chi-1})(t_{\varsigma+1}-\eta)^{\xi-1}d\eta,$$

$$D^{(\varsigma+1)}=\frac{\phi(1-\xi)}{AB(\phi)}t_{\varsigma}^{\phi-1}D_1(t_\varsigma,S(t_\varsigma),G)+\frac{\phi\xi}{AB(\xi)\Gamma(\xi)}$$

$$\Sigma_{\chi=2}^{\varsigma}D_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})t_{\chi-2}^{\phi-1}\times\int_{t_\chi}^{t_{\chi+1}}(t_{\varsigma+1}-\eta)^{\xi-1}d\eta+$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{\chi=2}^{\varsigma}\frac{1}{\varsigma t}[t_{\chi-1}^{\phi-1}D_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})-t_{\chi-2}^{\phi-1}$$

$$D_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]\times\int_{t_\chi}^{t_{\chi+1}}(\eta-t_{\chi-2})(t_{\varsigma+1}-\eta)^{\xi-1}d\eta+$$

$$\frac{\phi\xi}{AB(\xi)\Gamma(\xi)}\Sigma_{\chi=2}^{\varsigma}\frac{1}{2\varsigma t^2}[t_{\chi}^{\phi-1}D_1(t_\chi,S^\chi,E^\chi,I^\chi,Q^\chi,R^\chi,D^\chi)-2t_{\chi-1}^{\phi-1}$$

$$D_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})+\int_{t_\chi}^{t_{\chi+1}}t_{\chi-2}^{\phi-1}D_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},$$

$$I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]\times\int_{t_\chi}^{t_{\chi+1}}(\eta-t_{\chi-2})(\eta-t_{\chi-1})(t_{\varsigma+1}-\eta)^{\xi-1}d\eta.$$

Solving for the integral in the previous equation, we can carry out the following calculations and we get:

$$S^{(\varsigma+1)}=\frac{\phi(1-\xi)}{AB(\phi)}t_{\varsigma}^{\phi-1}S_1(t_\varsigma,S(t_\varsigma),G)$$

$$+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+1)}\Sigma_{\chi=2}^{\varsigma}t_{\chi-2}^{\phi-1}S_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})$$

$$\times[(\varsigma-\chi+1)^{\xi}-(\varsigma-\chi)^{\xi}]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+2)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi-1}^{\phi-1}S_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$-t_{\chi-2}^{\phi-1}S_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]$$

$$\times[(\varsigma-\chi+1)^{\xi}(\varsigma-\chi+3+2\xi)-(\varsigma-\chi)^{\xi}(\varsigma-\chi+3+3\xi)]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+3)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi}^{\phi-1}S_1(t_{\chi},S^{\chi},E^{\chi},I^{\chi},Q^{\chi},R^{\chi},D^{\chi})$$

$$-2t_{\chi-1}^{\phi-1}S_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$+t_{\chi-2}^{\phi-1}S_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})](P+H),$$

$$E^{(\varsigma+1)}=\frac{\phi(1-\xi)}{AB(\phi)}t_{\varsigma}^{\phi-1}E_1(t_{\varsigma},S(t_{\varsigma}),G)$$

$$+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+1)}\Sigma_{\chi=2}^{\varsigma}t_{\chi-2}^{\phi-1}E_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})$$

$$\times[(\varsigma-\chi+1)^{\xi}-(\varsigma-\chi)^{\xi}]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+2)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi-1}^{\phi-1}E_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$-t_{\chi-2}^{\phi-1}E_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]$$

$$\times[(\varsigma-\chi+1)^{\xi}(\varsigma-\chi+3+2\xi)-(\varsigma-\chi)^{\xi}(\varsigma-\chi+3+3\xi)]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+3)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi}^{\phi-1}E_1(t_{\chi},S^{\chi},E^{\chi},I^{\chi},Q^{\chi},R^{\chi},D^{\chi})$$

$$-2t_{\chi-1}^{\phi-1}E_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$+t_{\chi-2}^{\phi-1}E_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})](P+H),$$

$$I^{(\varsigma+1)}=\frac{\phi(1-\xi)}{AB(\phi)}t_{\varsigma}^{\phi-1}I_1(t_{\varsigma},S(t_{\varsigma}),G)$$

$$+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+1)}\Sigma_{\chi=2}^{\varsigma}t_{\chi-2}^{\phi-1}I_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})$$

$$\times[(\varsigma-\chi+1)^{\xi}-(\varsigma-\chi)^{\xi}]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+2)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi-1}^{\phi-1}I_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$-t_{\chi-2}^{\phi-1}I_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]$$

$$\times[(\varsigma-\chi+1)^{\xi}(\varsigma-\chi+3+2\xi)-(\varsigma-\chi)^{\xi}(\varsigma-\chi+3+3\xi)]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+3)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi}^{\phi-1}I_1(t_{\chi},S^{\chi},E^{\chi},I^{\chi},Q^{\chi},R^{\chi},D^{\chi})$$

$$-2t_{\chi-1}^{\phi-1}I_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$+t_{\chi-2}^{\phi-1}I_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})](P+H),$$

$$Q^{(\varsigma+1)}=\frac{\phi(1-\xi)}{AB(\phi)}t_{\varsigma}^{\phi-1}Q_1(t_{\varsigma},S(t_{\varsigma}),G)$$

$$+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+1)}\Sigma_{\chi=2}^{\varsigma}t_{\chi-2}^{\phi-1}Q_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})$$

$$\times[(\varsigma-\chi+1)^{\xi}-(\varsigma-\chi)^{\xi}]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+2)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi-1}^{\phi-1}Q_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$-t_{\chi-2}^{\phi-1}Q_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]$$

$$\times[(\varsigma-\chi+1)^{\xi}(\varsigma-\chi+3+2\xi)-(\varsigma-\chi)^{\xi}(\varsigma-\chi+3+3\xi)]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+3)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi}^{\phi-1}Q_1(t_{\chi},S^{\chi},E^{\chi},I^{\chi},Q^{\chi},R^{\chi},D^{\chi})$$

$$-2t_{\chi-1}^{\phi-1}Q_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$+t_{\chi-2}^{\phi-1}Q_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})](P+H),$$

$$R^{(\varsigma+1)}=\frac{\phi(1-\xi)}{AB(\phi)}t_{\varsigma}^{\phi-1}R_1(t_{\varsigma},G)$$

$$+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+1)}\Sigma_{\chi=2}^{\varsigma}t_{\chi-2}^{\phi-1}R_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})$$

$$\times[(\varsigma-\chi+1)^{\xi}-(\varsigma-\chi)^{\xi}]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+2)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi-1}^{\phi-1}R_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$-t_{\chi-2}^{\phi-1}R_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]$$

$$\times[(\varsigma-\chi+1)^{\xi}(\varsigma-\chi+3+2\xi)-(\varsigma-\chi)^{\xi}(\varsigma-\chi+3+3\xi)]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+3)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi}^{\phi-1}R_1(t_{\chi},S^{\chi},E^{\chi},I^{\chi},Q^{\chi},R^{\chi},D^{\chi})$$

$$-2t_{\chi-1}^{\phi-1}R_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$+t_{\chi-2}^{\phi-1}R_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})](P+H),$$

$$D^{(\varsigma+1)}=\frac{\phi(1-\xi)}{AB(\phi)}t_{\varsigma}^{\phi-1}D_1(t_{\varsigma},S(t_{\varsigma}),G)$$

$$+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+1)}\Sigma_{\chi=2}^{\varsigma}t_{\chi-2}^{\phi-1}D_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})$$

$$\times[(\varsigma-\chi+1)^{\xi}-(\varsigma-\chi)^{\xi}]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+2)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi-1}^{\phi-1}D_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$-t_{\chi-2}^{\phi-1}D_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})]$$

$$\times[(\varsigma-\chi+1)^{\xi}(\varsigma-\chi+3+2\xi)-(\varsigma-\chi)^{\xi}(\varsigma-\chi+3+3\xi)]+\frac{\xi(\varsigma t)^{\xi}}{AB(\xi)\Gamma(\xi+3)}$$

$$\Sigma_{\chi=2}^{\varsigma}[t_{\chi}^{\phi-1}D_1(t_{\chi},S^{\chi},E^{\chi},I^{\chi},Q^{\chi},R^{\chi},D^{\chi})$$

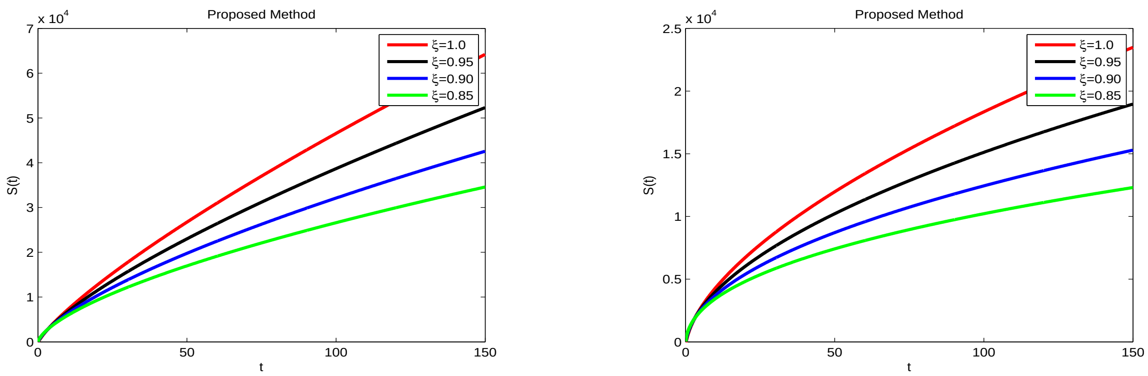
$$-2t_{\chi-1}^{\phi-1}D_1(t_{\chi-1},S^{\chi-1},E^{\chi-1},I^{\chi-1},Q^{\chi-1},R^{\chi-1},D^{\chi-1})$$

$$+t_{\chi-2}^{\phi-1}D_1(t_{\chi-2},S^{\chi-2},E^{\chi-2},I^{\chi-2},Q^{\chi-2},R^{\chi-2},D^{\chi-2})](P+H).$$

7. Simulation

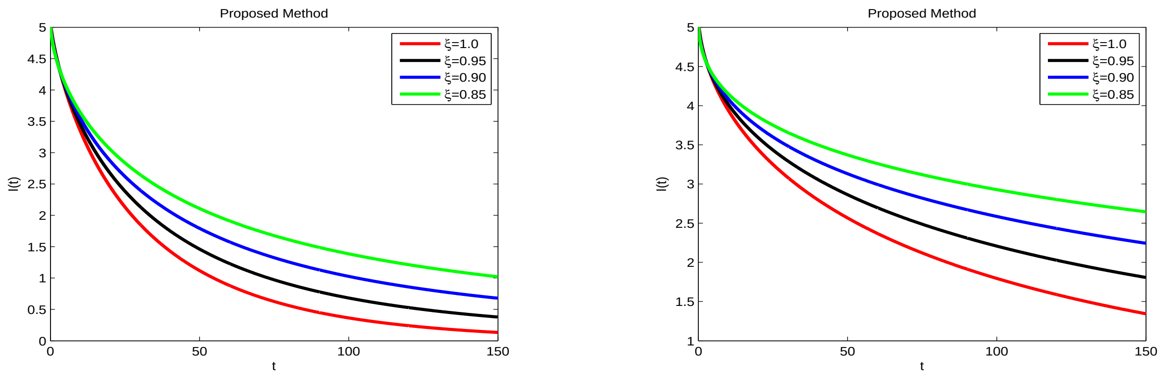
A mathematical analysis of citrus bacterial infection is presented, and by using non-integer parametric values, compelling results are obtained. By reducing the fractional values, the answer for P_S, P_E, P_I, P_R, S_v, and I_v, in figures 2-7 approaches the desired value. The numerical simulation for the fractional order citrus bacterial infection model is found using MATLAB code. The system's initial values are $P_S(0) = 0.70$, $P_E(0) = 0.55$, $P_I(0) = 0.43$, $P_R(0) = 0.68$, $S_v(0) = 0.70$ and $I_v(0) = 0.52$ for each of the sub-compartments. We show the graphical representation of the citrus bacterial infection model using the suggested numerical method in figures 27, and we compare the integer order result with the caputo fractional order result. The dynamics of citrus plants susceptible S_P and vector susceptible S_v caused by fungal and bacterial infection are shown in Figures 2 and 6, respectively. In these scenarios, all of the compartments rises, and after some time, they approached a steady position because to a rise in recovered. The dynamics of plants exposed P_E, plants infected P_I and plants recover P_R are shown in figures 3, 4 and 5, respectively. In these scenarios, all of the compartments rigidly sloped downward, as the recovered situation increased, the compartments approached a stable state. Additionally, as shown in figure 7, infected vector with and without treatment grows by decreasing the fractional values. It makes predictions on what this research will lead in the future and how we will be able to lower the number of sick plants and infected vectors that spread throughout the environment. When compared to traditional derivatives, the Caputo fractional operator yields better results for all sub-compartments at fractional derivatives. It is also suggested that as fractional values are reduced, the solutions for all compartments become more reliable and accurate.
Here, we computed theoretical results and evaluated their applicability using a sophisticated method. Figures 1-6 display the solutions for $S(t)$, $E(t)$, $I(t)$, $Q(t)$, $R(t)$, and $D(t)$.To bolster the validity of the theoretical results, we provide the following examples. The parameters that the suggested system employs are as follows:: $a = 1191$, $b = 0.98159$, $\alpha = 0.1$, $\beta = 0.7$, $c = 0.0006$, $e = 0.000001$, $f = 0.0007$, $g = 0.05$, $h = 0.015$, $k = 0.053$, $l = 0.012$ and the following initial circumstances; $S(0) > 0, E(0) > 0, I(0) > 0, Q(0) > 0, R(0) > 0, D(0) > 0$.

Dimensions 0.8 and 0.6 exhibit similar behavior with negligible effects; however, reducing dimensions yields more appropriate results, as shown in figures 2a5a and 2b5b, respectively. Using the suggested numerical technique, Nonetheless, worth more than the non-integer order styles that are currently in use. The behavior of the dynamics inside the special fractional parameters is displayed in the numerical results that are presented. Additionally, it is noted that recovery rises by decreasing the size and fractional values, as shown in figures 5a and 5b, respectively. It is concluded that by using combined early detection and acute and chronic stage investigation strategies, we can regulate COVID-19.



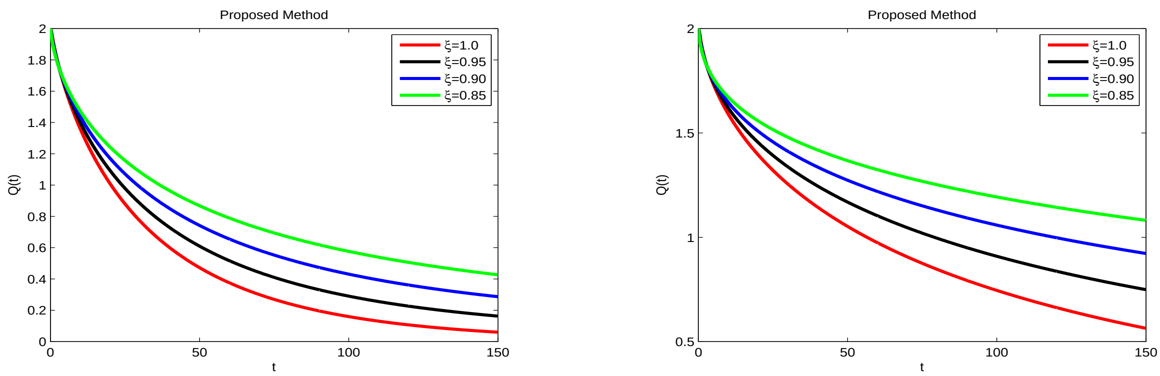
(a) Fractal Dimension 0.8 **(b)** Fractal Dimension 0.5

Figure 1: Dynamical behaviors of $S(t)$ with different fractional order.



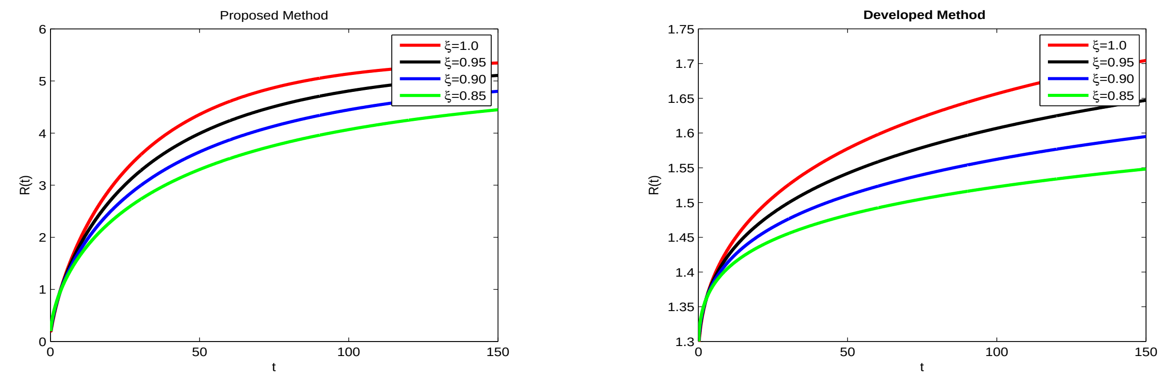
(a) Fractal Dimension 0.8 **(b)** Fractal Dimension 0.5

Figure 2: Dynamical behaviors of $I(t)$ with different fractional order.



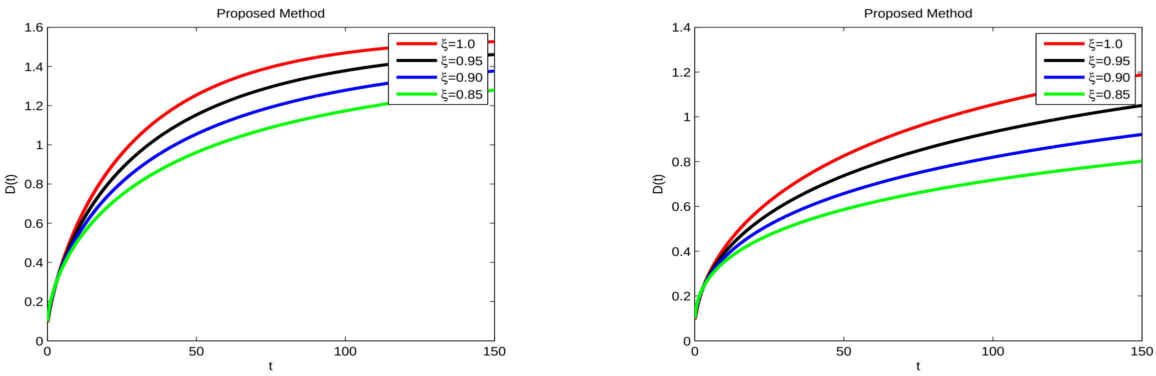
(a) Fractal Dimension 0.8 **(b)** Fractal Dimension 0.5

Figure 3: Dynamical behaviors of $Q(t)$ with different fractional order.



(a) Fractal Dimension 0.8 **(b)** Fractal Dimension 0.5

Figure 4: Dynamical behaviors of $R(t)$ with different fractional order.



(a) Fractal Dimension 0.8

(b) Fractal Dimension 0.5

Figure 5: Dynamical behaviors of $D(t)$ with different fractional order.

8. Conclusion

We examine a fractional order COVID-19 model that includes both acute and chronic stages in this study. For reliable insights, the Fractal-Fractional Operator (FFO) is utilized. Our inquiry explores the significant effects of the dangerous COVID-19 virus, highlighting the importance of early detection and prevention strategies worldwide. We verify the established system's stability by both quantitative and qualitative analysis, which is essential for comprehending its dynamic behavior over time. The model's behavior within confined parameters is outlined using local stability analysis, which is essential to understanding epidemic dynamics. We also look at the effectiveness of worldwide actions in reducing the spread of COVID-19 and validating their presence.Notably, strong immune responses and a combination of treatment strategies have led to a decrease in the number of COVID-19-infected people. The fractal-fractional Operator (FFO) was employed by us.maintain ongoing surveillance of viral transmission across many fractional values, producing trustworthy and practical findings. By using MATLAB-based numerical simulations, we are able to obtain a deeper understanding of the dynamics of COVID-19 control in real-world communities. Furthermore, the generated predictions provide a foundation for thoughtful future research,supporting the understanding of the mechanics of COVID-19 dissemination and providing early identification procedures.
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Conflict of interest: None

Refrences

[1] Chan, Jasper Fuk-Woo, et al. "Genomic characterization of the 2019 novel human-pathogenic coronavirus isolated from a patient with atypical pneumonia after visiting Wuhan." Emerging microbes and infections 9.1 (2020): 221-236.

[2] World Health Organization. Laboratory testing for coronavirus disease 2019 (COVID-19) in suspected human cases: interim guidance, 2 March 2020. No. WHO/COVID-19/laboratory/2020.4. World Health Organization, 2020.

[3] Lu, Hongzhou, Charles W. Stratton, and Yi-Wei Tang. "Outbreak of pneumonia of unknown etiology in Wuhan, China: The mystery and the miracle." Journal of medical virology 92.4 (2020): 401.

[4] D'Arienzo, Marco, and Angela Coniglio. "Assessment of the SARS-CoV-2 basic reproduction number, R0, based on the early phase of COVID-19 outbreak in Italy." Biosafety and health 2.2 (2020): 57-59.

[5] Aldila, Dipo, et al. "A mathematical study on the spread of COVID-19 considering social distancing and rapid assessment: The case of Jakarta, Indonesia." Chaos, Solitons and Fractals 139 (2020): 110042.

[6] Global, T., and O. Alert. ”Coronavirus disease 2019 (COVID19).” Geneva: world health organization (2020).

[7] Alnaser, Waheeb E., Mahmood Abdel-Aty, and Omar Al-Ubaydli. ”Mathematical prospective of coronavirus infections in Bahrain, Saudi Arabia and Egypt.” Inf Sci Lett 9.1 (2020): 51-64.

[8] Ming, Wai-Kit, Jian Huang, and Casper JP Zhang. ”Breaking down of healthcare system: Mathematical modelling for controlling the novel coronavirus (2019-nCoV) outbreak in Wuhan, China.” BioRxiv (2020).

[9] Nesteruk, Igor. ”Statistics-based predictions of coronavirus epidemic spreading in mainland China.” (2020).

[10] Batista, Milan. ”Estimation of the final size of the coronavirus epidemic by the logistic model.” medrxiv (2020): 2020-02.

[11] Okhuese, Victor Alexander. ”Mathematical predictions for COVID-19 as a global pandemic.” medrxiv (2020): 2020-03.

[12] Ivanov, Dmitry. ”Predicting the impacts of epidemic outbreaks on global supply chains: A simulation-based analysis on the coronavirus outbreak (COVID-19/SARS-CoV-2) case.” Transportation Research Part E: Logistics and Transportation Review 136 (2020): 101922.

[13] Li, Lixiang, et al. ”Propagation analysis and prediction of the COVID-19.” Infectious Disease Modelling 5 (2020): 282-292.

[14] Cooper, Ian, Argha Mondal, and Chris G. Antonopoulos. ”A SIR model assumption for the spread of COVID-19 in different communities.” Chaos, Solitons and Fractals 139 (2020): 110057.

[15] Fanelli, Duccio, and Francesco Piazza. ”Analysis and forecast of COVID-19 spreading in China, Italy and France.” Chaos, Solitons and Fractals 134 (2020): 109761.

[16] Petropoulos, Fotios, and Spyros Makridakis. ”Forecasting the novel coronavirus COVID-19.” PloS one 15.3 (2020): e0231236.

[17] Chintalapudi, Nalini, Gopi Battineni, and Francesco Amenta. ”COVID-19 virus outbreak forecasting of registered and recovered cases after sixty day lockdown in Italy: A data driven model approach.” Journal of Microbiology, Immunology and Infection 53.3 (2020): 396-403.

[18] Nafea, Marwan, et al. ”Hybrid PSO-Tuned PID and hysteresis-observer based control for piezoelectric micropositioning stages.” 2019 IEEE International Conference on Smart Instrumentation, Measurement and Application (ICSIMA). IEEE, 2019.

[19] Smethurst, Dominic P., and Hywel C. Williams. ”Are hospital waiting lists self-regulating?.” Nature 410.6829 (2001): 652-653.

[20] Meerschaert, Mark M., and Alla Sikorskii. Stochastic models for fractional calculus. Vol. 43. Walter de Gruyter GmbH and Co KG, 2019.

[21] Gomez-Aguilar, J. F., et al. ”Analysis of fractalfractional malaria transmission model.” Fractals 28.08 (2020): 2040041.

[22] Atangana, Abdon. ”Modelling the spread of COVID-19 with new fractal-fractional operators: can the lockdown save mankind before vaccination?.” Chaos, Solitons and Fractals 136 (2020): 109860.

[23] Khan, Hasib, et al. ”A fractional order HIV-TB coinfection model with nonsingular Mittag-Leffler Law.” Mathematical Methods in the Applied Sciences 43.6 (2020): 3786-3806.

[24] Shah, Kamal, et al. ”Semi-analytical study of Pine Wilt Disease model with convex rate under CaputoFebrizio fractional order derivative.” Chaos, Solitons and Fractals 135 (2020): 109754.

[25] Yousaf,M.,Zahir,S.,Riaz,M.,Hussain,S.M.,Shah,K.:Statistical analysis of forecasting COVID-19 for upcoming month in Pakistan.Chaos Solitons Fractals 138,109926(2020)

[26] Ud Din, Rahim, et al. ”Study of transmission dynamics of novel COVID-19 by using mathematical model.” Advances in Difference Equations 2020 (2020): 1-13.

[27] Shah, Kamal, Fahd Jarad, and Thabet Abdeljawad. ”On a nonlinear fractional order model of dengue fever disease under Caputo-Fabrizio derivative.” Alexandria Engineering Journal 59.4 (2020): 2305-2313.