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NATURE OF THE DIOPHANTINE EQUATION $4^X + 12^Y = Z^2$

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ABSTRACT- In this work, we discuss that the Diophantine equation $4^x + 12^y = z^2$has no non-negative integer solution where x, y and z are non-negative integers.

KEYWORDS- Exponential Diophantine Equation, Quadratic Congruence

I. INTRODUCTION

In 2011, Suvarnamani,Singtaand Chotchaisthit [15] showed that the two Diophantine equations$\mathbf{4^x + 7^y = z^2}$ and$\mathbf{4^x + 11^y = z^2}$ have no non-negative integer solution where x, y and z are non- negative integers.

In 2012, Chotchaisthit [14] showed that the Diophantine equation $\mathbf{4^x + p^y = z^2}$has no non-negative integer solution where x, y, z are non-negative integers and p is a positive prime.

In this paper, we show that the Diophantine equation of the type $4^x + 12^y = z^2$ has no non-negative integer's solution for all $\forall x, y \le 1$

II. PRELIMINARIES

Proposition2.1. (Catalan's conjecture)[1](3, 2, 2, 3) is a unique solution (a, b, x, y) for the Diophantine equation where a, b, x and y are integers such that min {a ,b, x, y}>1.

A. Lemma. [1, 15]

The Diophantine equation$4x + 1 = z^2$.The Diophantine equation $4^x + 1 = z^2$ has no non negative integer solution where x and z are non-negative integers.

B. lemma.

The Diophantine Equation$1 + 12^y$ have no non-negative integer solution where y and z are non-negative integers.

Proof:

Consider the non-negative integer y and z such that$1 + 12^y = z^2$-------- (I)

For y=0, it gives z^2=2 which is contract the assumption that z is a integer

Now for y ≥ 1, it gives $z^2 = 1 + 12^y \geq$ 1+12=13 and then z≥4

Now from equation (I), we have

$z^2 + 12^y = 1$By proposition (2.1) we get y=1, which is contradiction to the result $z^2 = 13$

Therefore, the Diophantine equation$1 + 12^y = z^2$does not contains any non negative solution.

III. RESULT ANALYSIS

A. Theorem

The Diophantine equation of the type$4^x + 12^y = z^2$, $\forall$ x,y >0 has no non – negative integer solution.

Proof:

Consider for $\forall$ x, y>0 such that$4^x + 12^y = z^2$

Now By lemma 2.2 and 2.3, we get x≥1 and also y≥1, it gives z is even .it implies that

z^2≡0 (mod3) orz^2=1(mod3)

But 4^x≡1 (mod3) and 12^x≡0 (mod3)

z^2≡1 (mod3), it leads to a contradiction.

Therefore, the equation of the type $4^x + 12^y = z^2$has no non-negative solution .for any choice of integers x, y and z

B. Corollary

For any positive integer t .Then the equation of the type $4^x + 12^y = a^{2t+2}\forall$ x, y, a>0 has no non-negative integers solution.

Proof:

Let us consider $4^x + 12^y = a^{2t+2}\forall$ x, y, t >0 has non – negative solution

Put $z = a^{t+1}$

Now $4^x + 12^y = a^{2t+2}$becomes$4^x + 12^y = z^2$, it leads to a contradiction

Therefore, the equation of the type $4^x + 12^y = a^{2t+2}$has no non-negative integer solutions.

C. Corollary

The Diophantine equation of the type $64^t + 10^y = z^2 \forall$ t, y, z > 0 has no non negative integer solution.

Proof:

Let us consider the equation $64^t + 12^y = z^2 \forall$ t, y, and z>0 has non –negative solution

Now the equation $64^t + 12^y = z^2$ can be written as

$$(4^{3^t}) + 12^y = z^2$$

$(4^{3t}) + 12^y = z^2$------------------------------- (II)

Let x=3t, Now the equation becomes (II)$4^x + 12^y = z^2$, this a contradiction to theorem 3.1

Therefore, the equation of the type $64^t + 12^y = z^2 \forall$ t, y, z>0 has no non-negative solution

IV. CONCLUSION

Generally, the exponential Diophantine equations of the type $p^x + q^y = z^2$ are helpful and wide application in the chemistry of chemical equation and network flow in data communication

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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