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EXPLORATION IN LINEAR ALGEBRA: A VIEWPOINT

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Abstract

The emphasis in linear algebra was changed to a matrix-oriented course focusing on applications and cutting down on the amount of time spent on idea abstraction as a result of the LACSG's recommendations. Although this change in emphasis benefits maths majors as well as non-majors, abstraction is being treated unfairly by being reduced to an 'also ran' when compared to applications. Since linear algebra's theory is so well-structured and extensive while requiring few mathematical prerequisites, it was chosen to be the first serious mathematics course in the undergraduate mathematics curriculum, according to Alan Tucker (1993) (p. 3). Even undergraduate mathematics majors who excelled in calculus I and II face difficulties in linear algebra. Since it is the first subject in which students are expected to prove theorems, it is crucial to their development as conjecturers and writers of cogent proofs. According to Tucker, "a practitioner or researcher in most areas of pure and applied mathematics needs to have a solid understanding of finite vector spaces, linear transformations, and their extensions to function spaces". Determinants-more especially, the formulation and validation of the determinant's fundamental properties-are one area that is being underemphasized.

Keywords: Linear Algebra, Architecture, Maths, Modern.

Introduction

Calculus has been the main area of reform in collegiate mathematics programmes. In the discipline of linear algebra, however, a considerably more subdued reform effort has been going on for the past five years. In response to the worry that "the linear algebra curriculum at many schools does not adequately address the needs of the students it attempts to serve," the Linear Algebra Curriculum Study Group (LACSG) was established in January 1990 (Porter, 1993, p. 41). They discovered that although there had been a sharp rise in demand for the course from "client disciplines such as engineering, computer science, operations research, economics, and statistics" (Porter, 1993, p. 41), the format and content had stayed the same. The typical introductory linear algebra course has been restructured as a result of their concerns about the topics covered, the emphasis on abstraction of concepts at the expense of real-world applications, and the apparent lack of technology used by disciplines that utilise the concepts of linear algebra.

It is odd, considering the evolution of matrix theory over the years, that this focus is now diverging from determinant research. Tucker (1993) claims that Leibniz utilised determinants-not matrices-150 years before J. J. Sylvester first used the name in 1848. Determinants originated from the study of coefficients of systems of linear equations. The finding det(AB) = det(A) det(B) provided the essential connection between the recently created matrix theory and the long-standing study of determinants. One of the fundamental qualities whose development and verification are being removed from the curriculum is this same result.

All educational levels (K–12) incorporate the four core areas of problem solving, communication, reasoning, and connections, as outlined in the NCTM Curriculum Standards (1989). These themes are also present in their learning objectives, which state that students should "learn to value mathematics, to become confident in their mathematical abilities, to solve mathematical problems, to communicate mathematically, and to reason mathematically" (p. 5). "These goals imply that students should be exposed to numerous and varied interrelated experiences that foster a value for the mathematical enterprise, the development of mathematical habits of mind, and an appreciation of the role of mathematics in human affairs," the statement continues. "They also call for students to read, write, and discuss mathematics; they should conjecture, test, and build arguments about the validity of a conjecture; and they should be encouraged to explore, guess, and even make and correct errors so that they gain confidence in their ability to solve complex problems." It is possible to accomplish these objectives by studying linear algebra. Nevertheless, an applications-based approach is taking precedence over the material, which is rich in mathematical investigations, in the curriculum.

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Students' experience and maturity in mathematics can be improved by investigating characteristics that link ideas in linear algebra. It's surprising how slowly technology adoption has spread throughout college. But studies reveal that it enhances attitudes and academic performance in students. According to Peck et al. (1994), there was a considerable improvement in student achievement not only in the technology-enhanced course but also in succeeding non-technical courses. They discovered that by allowing pupils to concentrate on comprehending the issues and applying mathematics, technology use "allowed the students to develop their mathematical skills" (Peck, 1994, p.6). Quesada and Maxwell (1994) looked at the benefits and drawbacks of teaching pre-calculus with graphing calculators. When compared to students in a typical course utilising scientific calculators, they found that the usage of graphing calculators increased student achievement. Students in the experimental group indicated on a survey that they spent more time studying, had more freedom to explore, and had a better understanding of the material. Research by Guckin and Morrison (1991), Stiff et al. (1992), Peck et al. (1992), Quesada and Maxwell, and others clearly show that when students are taught using technology, they respond with a greater level of success and an increase in favourable attitudes. All of these scholars acknowledge, however, that their ability to teach students in a more conceptual, constructivist way and to add real-world applications that give themes significance comes from their use of technology.

Compared to the 200 years that calculus has been taught, linear algebra is a relatively recent addition to the undergraduate mathematics curriculum. That being said, this does not lessen its importance in a mathematics programme. Actually, calculus frequently plays the role of a service course in linear algebra, a necessity that is growing quickly for other degree programmes. Applications for the linear algebraic methods can be found in a wide range of disciplines, including economics, engineering, physical science, social science, and archaeology, to mention a few. Many teachers have chosen to focus more on the practical applications of linear algebra rather than eliminating significant conceptual abstractions due to the inflow of students majoring in different degree programmes. There has also been a significant dilution in abstraction as a result of other departments opting to teach linear algebra in a different way. Math departments frequently decide to give "them what they want" and tone down their curriculum in response to "turf protection" in order to keep student enrollment high.

It is regrettable to say the least that theory was reduced to a supporting role in the development of linear algebra. An essential course in an undergraduate mathematics programme is linear algebra. For many of the pupils, it is their first subject where they face challenges in maths. Future mathematical research in group and ring theory, combinatorics, and analysis will all build upon linear algebra. Therefore, in order to create further mathematical knowledge, students who are finishing linear algebra need to have a solid mathematical background and grasp of the relevant ideas. An instructor's challenge in each mathematics course is to strike a balance between the beauty and usefulness of mathematics. It is important to provide students with the chance to investigate topics in linear algebra that are abundant in linkages and applications.

Literature Review

In the Cobani (2021) year; Inverse scattering is currently receiving a great deal of attention from mathematicians who work with the theory of partial differential equations, and this field of study is consistently making progress. An inhomogeneous medium's ability to scatter light is highly problematic. We study the transmission eigenvalue matter corresponding to a new scattering problem in this paper. Unlike any other previously solved internal problem, the boundary conditions of this new scattering problem are unique. More precisely, we consider the case when the variation of the track of both fields is proportional to the normal derivative of the field, as opposed to dictating the difference Cauchy data on the boundary, which is the conventional approach for the problem. This is because the problem's classical version requires the existence of the difference Cauchy data on the border. Typical concerns related to the transmission eigenvalue problem (TEP) include the existence of the transmission eigenvalues, their discreteness, and the Fredholm condition as well as solvability. We discuss and offer solutions to all of these queries and problems related to an interior gearbox problem involving an inhomogeneous medium in this article. We apply the variational technique and a very important theorem regarding the existence of transmission eigenvalues to conclude that they do exist. Combined, these two instruments enable us to demonstrate the existence of transmission eigenvalues. The eigenvalues of the gearbox must be discrete in order to demonstrate that the problem can be solved because the interior gearbox scenario satisfies the Fredholm Alternative. This is a crucial phase in the procedure. Discreteness is necessary to guarantee that the reconstruction methods-which include linear sampling-succeed in reconstructing the scatterer within an inhomogeneous medium. This is due to the fact that discreteness a practical need. Transmission eigenvalues are significant because they provide information about the composition of inhomogeneous medium. This is among the factors that make their existence so crucial. Transmission eigenvalues can be used to determine an object's index of refraction by combining them with observational data, as demonstrated by recent studies. This is on top of the theoretical importance of transmission eigenvalues in the context of reconstruction leading to inverse scattering theory and uniqueness. This is because it was discovered-a noteworthy discovery-that transmission eigenvalues could be inferred from recorded far field data. This serves as the basis for the argument. The interior problem is the most significant issue in inverse scattering. We will focus our efforts on solving the interior problem, which is the most significant obstacle offered by inverse scattering, after thoroughly examining the topic of the well-posedness underlying the direct issue, which must be completed before proceeding to the inverse issue. It is related to the transmission eigenvalues that the interior problem in an inhomogeneous medium is difficult to solve. The most concerning is the question of whether transmission eigenvalues are real or not.

2021's Diao; This research investigates the intrinsic geometrical patterns for conductive transmission eigenfunctions. The geometric properties of internal transmission eigenfunctions were investigated first. It is shown which interior transmission eigenfunction has to have a locally vanishing value near a domain corner with an interior angle less than in two different examples. We significantly generalise and expand upon these findings in several ways. Firstly, we consider the conductive transmission eigenfunctions which also include the internal transmission eigenfunctions as a particular case. The findings are integrated into the geometric structures that this paper created to describe eigenfunctions of conductive transmission. Second, as long as the interior angle of the corner is not when the conductive transmission eigenfunctions satisfy specific approximation qualities of Herglotz functions, the vanishing characteristic of the eigenfunctions can be constructed for any corner. When the corner's inner angle is not, this can be done. That is to say, if the fundamental conductive transmission eigenfunctions can be approximated using a series of Herglotz functions under reasonable approximation rates, the disappearing feature will hold true as long as the corner singularity does not degrade. This is due to the fact that the sequence of Herglotz functions provides a more accurate approximation of the eigenfunctions than the Herglotz function sequence. Third, in the current study for the conductive transmission eigenfunctions, the regularity constraints for the interior transmission eigenfunctions are significantly relaxed. This was carried out to increase the study's applicability to actual situations. We have to develop technically new ways to obtain the geometric attributes of the conductive transmission eigenfunctions, and the analysis associated with such approaches is considerably more complex. In summary, we identify a unique recovery origin for the reverse problem related to transverse electromagnetic dispersion calculation through a single far-field evaluation within the simultaneous determination of the polygonal conductive obstacle and its surface conductive variable. This is an intriguing and practical application of the obtained geometric outcomes. The simultaneous determination of a polygonal conductive difficulty and its surface conductive variable is used to achieve this.

Analysis

Linear algebra is the study of mappings between vector spaces that maintain the vector-space structure, much like in the theory of other algebraic structures. A linear transformation, also known as a linear map, linear mapping, or linear operator, is a map given two vector spaces V and W over a field F.

$$T : V \to W$$

that works with scalar multiplication and addition:

$$T(u+v) = T(u) + T(v), \quad T(av) = aT(v)$$

for each scalar a ∈ F and any vectors u,v ∈ V.

Furthermore, given any vectors u, v ∈ V and scalars a, b ∈ F:

$$T(au+bv) = T(au) + T(bv) = aT(u) + bT(v)$$

Two vector spaces are said to be isomorphic when there is a bijective linear mapping between them, meaning that each vector in the second space is associated with exactly one in the first. From the perspective of linear algebra, two isomorphic vector spaces are "essentially the same" since an isomorphism maintains linear structure. Finding out if a mapping is an isomorphism or not is a fundamental question in linear algebra, which can be addressed by determining whether the determinant is nonzero. Finding a mapping's range (or image) and the set of items that are mapped to zero-referred to as the mapping's kernel-are of importance to linear algebraists if the mapping is not an isomorphism.

The meaning of linear transformations is geometric. For instance, conventional planar mappings that maintain the origin are represented as 2 × 2 real matrices.

1. Subspaces, Span, and Basis

Once more, linear algebra is interested in subsets of vector spaces that are also vector spaces; these subsets are referred to as linear subspaces, and their theories are analogous to those of other algebraic objects. Important examples of subspaces are the range and kernel of a linear mapping, which are sometimes known as the null space and the range space, respectively. Forming a subspace by using a linear combination of a set of vectors $v_1, v_2,..., v_k$ is another crucial method:

$$a_1v_1 + a_2v_2 + \cdots + a_kv_k,$$

where the scalars are $a_1, a_2,..., a_k$. Their span, a subspace, is the set of all linear combinations of the vectors $v_1, v_2,..., v_k$.

The zero vector of V is a linear combination of any system of vectors with all zero coefficients. These vectors are linearly independent if this is the only way to represent the zero vector as a linear combination of v1, v2,..., vk. If a vector w in a set of vectors that spans a space is a linear combination of other vectors (i.e., the set is not linearly independent), then removing w from the set will not change the span. Because a linearly independent subset will exist and span the same subspace, a collection of linearly dependent vectors is therefore redundant. Thus, a linearly independent set of vectors spanning a vector space V-which we refer to as a basis of V-is the main object of our concern. A basis can be extended to any linearly independent set of vectors in V, and any set of vectors spanning across V contains a basis. It turns out that every vector space has a basis if we accept the axiom of choice; nevertheless, this basis might not even be constructible and might even be unnatural. As an example, the real numbers have a basis, which is viewed as a vector space over the rationals; nevertheless, no explicit basis has been built.

A vector space V's dimension is the set of all two bases that have the same cardinality. The vector space dimension theorem provides a clear definition of a vector space's dimension. V is referred to as a finite-dimensional vector space if each basis of V contains a finite number of elements. Dim U ≤ dim V if V is finite-dimensional and U is a subspace of V. If V has U1 and U2 as subspaces, then

$$\dim(U_1 + U_2) = \dim U_1 + \dim U_2 - \dim(U_1 \cap U_2)$$

Frequently, consideration is limited to vector spaces with a finite dimension. All vector spaces of the same dimension are isomorphic, according to a basic theorem of linear algebra, providing a simple method for characterising isomorphism.

2. Matrix Theory

One can create a coordinate system in V using a certain basis $\{v_1, v_2,..., v_n\}$ of V: the vector with coordinates $(a_1, a_2,..., a_n)$ is the linear combination of

$$a_1v_1 + a_2v_2 + \cdots + a_nv_n.$$

While the linear independence of $v_1, v_2,..., v_n$ ensures that these coordinates are unique (i.e., there is only one linear combination of the basis vectors that is equal to v), the constraint that $v_1, v_2,..., v_n$ span V ensures that each vector v can be assigned coordinates. In this manner, V may be identified with the coordinate n-space F^n once a basis of a vector space V over F has been selected. According to this identification, the coordinate vector addition and multiplication of vectors in V correspond to the coordinate vector addition and multiplication of vectors in Fn. Moreover, if V and W are vector spaces over F that are n- and m-dimensional, respectively, and if the bases of V and W are fixed, then any linear transformation T: $V \to W$ may be represented as the matrix of T with regard to these bases, an m × n matrix A having entries in the field F. Similarity between two matrices is defined as encoding the same linear transformation in distinct bases. The study of matrices, which are concrete entities, takes the place of the axiomatically defined study of linear transformations in matrix theory. This key method sets linear algebra apart from theories of other algebraic structures, which are typically not amenable to such a detailed parameterization.

The coordinate n-space $\mathbf{R}^n$ and a general finite-dimensional vector space V differ significantly. A vector space V usually lacks a standard basis, whereas $\mathbf{R}^n$ has a standard basis $\{e_1, e_2,..., e_n\}$. Despite this, there are numerous bases that can be chosen from, all of which have an equal number of elements, equivalent to the dimension of V.

The computation of determinants, a key idea in linear algebra, is one important use of matrix theory. Although determinants can be defined without regard to bases, they are typically introduced through a particular mapping representation; the determinant's value is independent of the basis. It turns out that the inverse of a mapping exists only if the determinant (i.e., every non-zero real or complex integer) does. The null space is nontrivial if the determinant is zero. A systematic method of determining if a set of vectors is linearly independent (we write the vectors as the columns of a matrix; if the determinant of that matrix is zero, the vectors are linearly dependent) is one of the various uses for determinants. While determinants can also be used to solve linear equation systems (see Cramer's rule), Gaussian elimination is a faster approach in practical applications.

Conclusion

For those who would like a copy, there is a module that walks students through the development of the concept of determinants-a topic that has been less stressed recently in most introductory linear algebra courses-and focuses on connecting the determinant of a matrix with other important concepts in linear algebra, like inverses, gaussian elimination, eigenvalues, and eigenvectors. (The postal and email addresses can be found at the conclusion of this article.) The study is included in the module, and a synopsis of the module's contents is provided below.

This curriculum module's overarching goals are to give students the chance to investigate, hypothesise, and prove hypotheses; engage with peers; and see examples that demonstrate how well appropriate technology use can be used to explore and develop mathematical concepts.

In this, the people learn about the determinant as a function that transfers the set of real numbers to a subset of all matrices having real number entries. Prior to anything else, the students need to explain the subset of matrices that make up the function's domain. Using a graphing calculator (TI-81, 82, or 85), users may determine the determinant of any matrix given a few instances of matrices. The absence of a determinant in a matrix indicates that it is beyond the function's scope. They will utilise the calculator to investigate basic cases (such as a 2 x 2 case) after describing the domain of the determinant function. The relationship between the determinant and the matrix's entries will be revealed to the student. The students will investigate specific matrix types (such as triangular or diagonal) in order to come up with a technique for determining the determinants for these types of matrices once they have developed a "formula" for determining the determinant of a 2 x 2 matrix.

Conflicts of Interest

The authors declare that there are no significant competing financial, professional, or personal interests that might have influenced the performance or presentation of the work described in this manuscript.

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